# Math 1120F - Exam 1

Name: KEY

Friday, November 7, 2014
Time: 50 minutes
Instructor: Brittany Cuchta

**Instructions:**

- Do not open the exam until I say you may.
- All cell phones and other electronic noisemaking devices must be turned off or completely silenced (i.e., not on vibrate) for the duration of the exam.
- **No calculators** are allowed on the exam.
- The exam *must* be taken in pencil. Using a pen on the exam will result in the loss of points.
- Failure to follow directions specific to a problem will result in the loss of points.
- Circle or box your final answer where appropriate. Put your final answer in the provided space when available. Failure to do so will result in points being deducted.
- Show **all** work. Full credit will only be given if work is shown which **fully and clearly** justifies your answer. I reserve the right to not grade a problem which I cannot read.
- Answers must be exact (like $\sqrt{2}$), not approximate (like 1.414), unless a problem specifically indicates otherwise.
- All final answers must be simplified unless otherwise specified. **Rationalization is not required unless otherwise specified.**
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

| Page: | 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|---|
| Points: | 26 | 15 | 22 | 12 | 75 |
| Score: | | | | | |

1. (8 points) Circle true or false for the following questions. Partial credit will not be given.

   (a) True False : If $f$ is a function and $f^{-1}$ is its inverse, then $(f \circ f^{-1})(y) = y$ and $(f^{-1} \circ f)(x) = x$.

   (b) True False : The domain of $y = a^x$ is all real numbers.

   (c) True False : The domain of $y = \log_a x$ is all real numbers.

   (d) True False : If $y = x^a$ then $\log_a y = x$.

2. (10 points) Complete the sentence or equation with the correct answer by filling in the blank. Partial credit will not be given.

   (a) $\log_a xy =$ $\log_a x + \log_a y$

   (b) $\log_a \frac{x}{y} =$ $\log_a x - \log_a y$

   (c) $\log_a a^r =$ $r$

   (d) $a^{\log_a M} =$ $M$

   (e) In order for a function to have an inverse, it must be one-to-one (a special property).

3. (8 points) If $f(x) = \frac{x+1}{x-3}$ and $g(x) = \frac{x-1}{2x}$, find $(f \circ g)(x)$ and give its domain in set notation.

$$(f \circ g)(x) = \frac{\frac{x-1}{2x} + 1}{\frac{x-1}{2x} - 3} = \frac{\frac{x-1+2x}{2x}}{\frac{x-1-6x}{2x}} = \frac{3x-1}{-5x-1}$$

$D_f = \{x \mid x \neq 3\}$

$D_g = \{x \mid x \neq 0\}$

$D_{f \circ g} = \{x \mid x \neq 0, x \neq -\frac{1}{5}\}$

Domain ?

$g(x) = 3$

$\frac{x-1}{2x} = 3$

$x - 1 = 6x$

$-5x = 1 \Rightarrow x = -\frac{1}{5}$

$(f \circ g)(x) =$ $\frac{3x-1}{-5x-1}$

Domain: $\{x \mid x \neq 0, x \neq -\frac{1}{5}\}$

4. (10 points) Find the inverse of the following function. Give the domain and range of the inverse function in set notation.

$$G(x) = \frac{3x}{x+5}$$

$$y = \frac{3x}{x+5}$$

$$D_G = \{x \mid x \neq -5\} = R_{G^{-1}}$$

$$yx + 5y = 3x$$

$$D_{G^{-1}} = \{y \mid y \neq 3\}$$

$$yx - 3x = -5y$$

$$x(y-3) = -5y$$

$$x = \frac{-5y}{y-3}$$

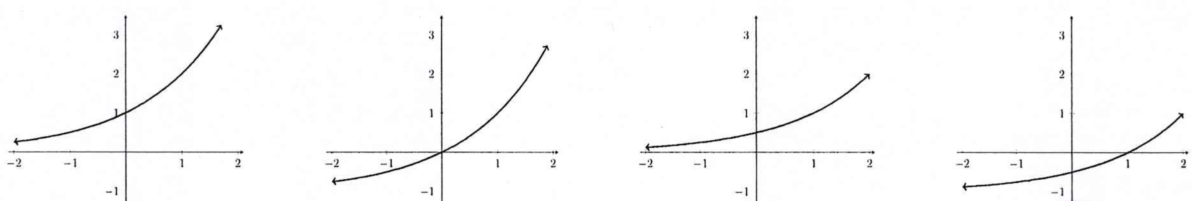
$G^{-1}$ : $-\dfrac{5y}{y-3}$

Domain of $G^{-1}$ : $\{y \mid y \neq 3\}$

Range of $G^{-1}$ : $\{x \mid x \neq -5\}$

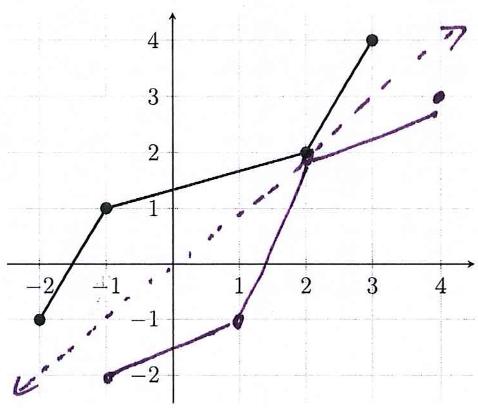
5. (2 points) The equation of an exponential function is given below. Choose the graph that best represents the function. Partial credit will not be given.

$$h(x) = 2^{x-1} - 1$$



A B C D

6. (3 points) Graph the inverse of the given function on the same grid.



7. Solve the following equations. Be sure to list *all* powers as factors in answers with logarithms.

(a) (8 points) $\log_2 x + \log_2 (x-2) = 3$

$\log_2 x + \log_2(x-2) = 3$

$\log_2(x(x-2)) = 3$

$x^2 - 2x = 2^3$

$x^2 - 2x - 8 = 0$

$(x+2)(x-4) = 0$

$x = -2, \quad x = 4$

$\log_2(-2)$ is undefined $\therefore$ the only solution is $x = 4$

Solution: $x = 4$

(b) (6 points) $5^{2x} = 25^{x^2}$

$5^{2x} = 25^{x^2}$

$5^{2x} = (5^2)^{x^2}$

$5^{2x} = 5^{2x^2}$

$2x = 2x^2$

$2x - 2x^2 = 0$

$x(2-2x) = 0$

$x=0$ or $x = +1$.

Solution: $x = 0, 1$.

(c) (8 points) $3^{2x} + 3^x = 2$

$3^{2x} + 3^x = 2$

$3^{2x} + 3^x - 2 = 0$

$(3^x)^2 + 3^x - 2 = 0$

Let $u = 3^x$. Then

$u^2 + u - 2 = 0$

$(u-1)(u+2) = 0$

$u = 1$ or $u = -2$

$3^x = 1$ $\qquad$ $3^x = -2$

$x = 0$ $\qquad$ DNE

Solution: $x = 0$

*note that taking a logarithm with a different base will get you the same numerical answer.

(d) (6 points) $3^{1-2x} = 4^x$

$$3^{1-2x} = 4^x$$

$$\log(3^{1-2x}) = \log(4^x)$$

$$(1-2x)\log 3 = x\log 4$$

$$\log 3 - 2x\log 3 = x\log 4$$

$$\log 3 = x\log 4 + 2x\log 3$$

$$\log 3 = x(\log 4 + 2\log 3)$$

$$x = \frac{\log 3}{\log 4 + 2\log 3}$$

Solution: $\frac{\log 3}{\log 4 + 2\log 3}$

8. (5 points) If $\log a = 2$, $\log b = 4$ and $\log c = 1$, evaluate the following. Your answer should be an integer.

$$\log\left[\left(\frac{a^2\sqrt[4]{b}}{c^3}\right)^2\right]$$

$$\log\left[\left(\frac{a^2\sqrt[4]{b}}{c^3}\right)^2\right] = 2\log\left(\frac{a^2\sqrt[4]{b}}{c^3}\right) = 2\log(a^2\sqrt[4]{b}) - 2\log(c^3)$$

$$= 2\log(a^2) + 2\log(\sqrt[4]{b}) - 2\log(c^3)$$

$$= 4\log a + \frac{1}{2}\log b - 6\log c$$

$$= 4(2) + \frac{1}{2}(4) - 6(1)$$

$$= 8 + 2 - 6 = \boxed{4}$$

9. (1 point) Realistically estimate how well you feel you feel you performed on the exam (using letter grades or number grades).