# Math 1120F - Exam 4

Name: KEY

Friday, December 5, 2014
Time: 35 minutes
Instructor: Brittany Cuchta

**Instructions:**

- Do not open the exam until I say you may.
- All cell phones and other electronic noisemaking devices must be turned off or completely silenced (i.e., not on vibrate) for the duration of the exam.
- **No calculators** are allowed on the exam.
- The exam *must* be taken in pencil. Using a pen on the exam will result in the loss of points.
- Failure to follow directions specific to a problem will result in the loss of points.
- Circle or box your final answer where appropriate. Put your final answer in the provided space when available. Failure to do so will result in points being deducted.
- Show **all** work. Full credit will only be given if work is shown which **fully and clearly** justifies your answer. I reserve the right to not grade a problem which I cannot read.
- Answers must be exact (like $\sqrt{2}$), not approximate (like 1.414), unless a problem specifically indicates otherwise.
- All final answers must be simplified unless otherwise specified. **Rationalization is not required unless otherwise specified.**
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

| Page: | 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|---|
| Points: | 21 | 6 | 10 | 13 | 50 |
| Score: | | | | | |

1. (6 points) Circle true or false for the following questions. Partial credit will not be given.

(a) True **False** : The graph of a rational function never intersects an asymptote.

(b) True **False** : Every polynomial function of degree three has exactly three real zeros.

(c) True **False** : A polynomial function of degree 4 with real coefficients could have 3, $1-2i$, $1+2i$, and $5-i$ as its zeros.

2. (6 points) Complete the following statements by filling in the blank with the correct answer. Note that partial credit will not be given.

(a) When a polynomial function $f(x)$ is divided by $x-c$ the remainder is $f(c)$.

(b) The graph of the function $g(x) = 4x^4 + 2x^3 + x$ will behave like $x^4$ for large values of $|x|$.

(c) If, as $x \to \infty$ or as $x \to -\infty$, the values of $R(x)$ approach some fixed number $L$, then the line $y = L$ is a horizontal asymptote of the graph of $R$.

3. (4 points) The graph of the polynomial $r(x) = x^3(x+4)(2x-1)^4$

(a) touches the $x$-axis at $x =$ $\frac{1}{2}$

(b) crosses the $x$-axis at $x =$ $0, -4$

4. (5 points) Find the domain of the following function. Give your answer in interval notation.

$$\sqrt{\frac{x-3}{x+4}}$$

$\frac{x-3}{x+4} \geq 0$

zeros: 3

asymp: $-4$

$(-\infty,-4)$ $-4$ $(-4,3)$ $3$ $(3,\infty)$

$-5$ $0$ $4$

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Domain: $(-\infty,-4) \cup [3,\infty)$

5. Given the function $T(x) = \dfrac{x^2 - 6x + 9}{x^2} = \dfrac{(x+3)^2}{x^2}$

(a) (1 point) State the domain. Give your answer in set notation.

Domain: $\{x \mid x \neq 0\}$

(b) (2 points) List the $x$ and $y$ intercepts. If there is none, state so in the provided space.

x-int: $0 = (x-3)^2$

$x = 3$

y-int: DNE

$x$-intercept(s): $(3, 0)$

$y$-intercept: DNE

(c) (3 points) State on which intervals the graph is above the $x$-axis and below the $x$-axis. Use the table method discussed in class.

| $(-\infty, 0)$ | 0 | $(0, 3)$ | 3 | $(3, \infty)$ |
|---|---|---|---|---|
| $-1$ | | $1$ | | $4$ |
| $\dfrac{+}{+}$ | | $\dfrac{+}{+}$ | | $\dfrac{+}{+}$ |

Above Axis: $(-\infty, \infty)$

Below Axis: none

(d) (4 points) Find all vertical, horizontal or oblique asymptotes. If the result doesn't exist, state so in the space provided. If the function crosses the asymptote, state where. If it does not cross, state so.

$$\begin{array}{r|l} & 1 \\ x^2 & x^2-6x+9 \\ & -x^2 \\ \hline & -6x+9 \end{array}$$

$$\frac{(x-3)^2}{x^2} = 1$$

$$x^2-6x+9 = x^2$$

$$9 = 6x$$

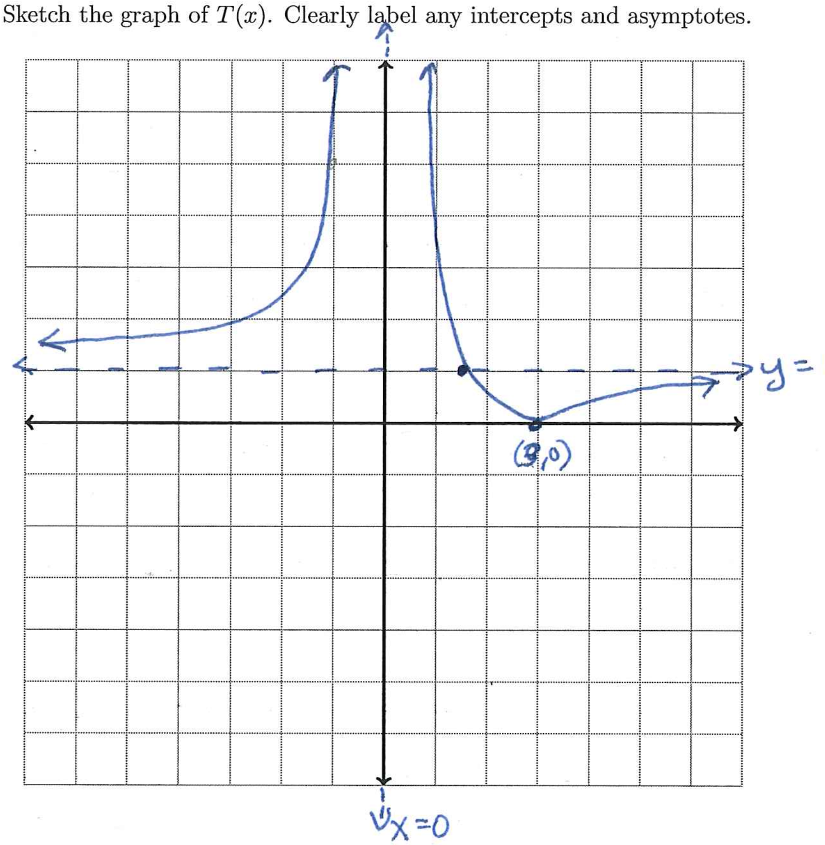
$$x = \frac{3}{2}$$

Vertical Asymptote(s): $x=0$ Crosses at: DNE

Horizontal Asymptote: $y=1$ Crosses at: $x=\frac{3}{2}$

Oblique Asymptote: DNE Crosses at: DNE

(e) (6 points) Sketch the graph of $T(x)$. Clearly label any intercepts and asymptotes.



6. (4 points) Construct a polynomial $f(x)$ with the following characteristics. Leave the polynomial in factored form.

   (a) Zeros: 2 (multiplicity 1), -1 (multiplicity 3), 0 (multiplicity 1)

   (b) Degree: 5

   (c) Contains the point: (1, 8)

$f(x) = a(x-2)(x+1)^3 x$

$8 = a(1-2)(2)^3(1)$

$8 = a(-1)(8)$

$8 = -8a$

$a = -1$

Polynomial $f(x) =$ $-(x-2)(x+1)^3 x$

7. For the polynomial function $H(x) = x^3 - x^2 + 4x - 4$:

   (a) (2 points) Is $x = 1$ a zero for $H(x)$? Show work and circle your answer.

$(1)^3 - (1)^2 + 4 - 4 = 0$

By remainder theorem, $x=1$ is a root.

Circle One: Yes No

   (b) (2 points) Use the **Rational Root Theorem** to find all **real** zero(s) of $H(x)$. Using any other method will result in no points being given.

from (a) we know $x=1$ is a root. possible roots via RRT:

$p = \pm 1, \pm 2, \pm 4 \Rightarrow \frac{p}{q} = \pm 1, \pm 2, \pm 4$

$q = \pm 1$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 4 & -4 \\ & \downarrow & 1 & 0 & 4 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$x^2 + 4$

This has no real solutions

Real Zero(s): $x = 1$

   (c) (5 points) Find all **complex** zeros of $H(x)$.

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

Complex Zero(s): $\pm 2i$