Math 1120F - Fall 2014
Worksheet 13 - §2.4

Name(s): KEY

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Work with partners in groups of 2-4. **This is required.**

1. Write the standard form of the equation and the general form of the equation of the circle with radius r and center (h, k).

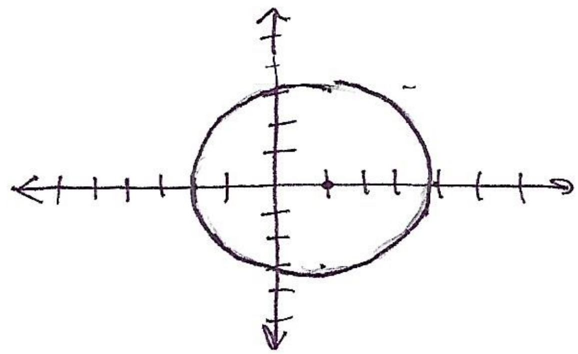
   (a) $r = 3$; $(h, k) = (1, 0)$

   $(x-1)^2 + (y-0)^2 = 9$

   $\boxed{(x-1)^2 + y^2 = 9}$

   $x^2 - 2x + 1 + y^2 = 9$

   $\boxed{x^2 - 2x + y^2 - 8 = 0}$



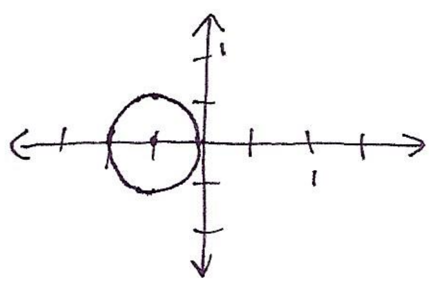
   (b) $r = \frac{1}{2}$; $(h, k) = (0, -\frac{1}{2})$

   $(x-0)^2 + (y+\frac{1}{2})^2 = \frac{1}{4}$

   $\boxed{x^2 + (y+\frac{1}{2})^2 = \frac{1}{4}}$

   $x^2 + y^2 + y + \frac{1}{4} = \frac{1}{4}$

   $\boxed{x^2 + y^2 + y = 0}$

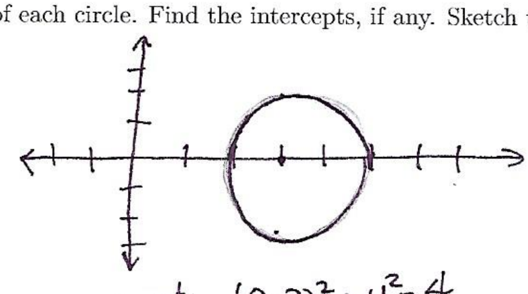


2. Find the center and the radius of each circle. Find the intercepts, if any. Sketch the circle.

   (a) $2(x-3)^2 + 2y^2 = 8$

   $(x-3)^2 + y^2 = 4$

   Center: $(3, 0)$
   Radius: 2



   Intercepts: $(3, 1)$, $(3, 5)$

   x-int: $(x-3)^2 = 4$

   $x - 3 = \pm 2$, $x = 3 \pm 2$

   y-int: $(0-3)^2 + y^2 = 4$

   DNE $y^2 = -5$

   (b) $x^2 + y^2 + 4x + 2y - 20 = 0$

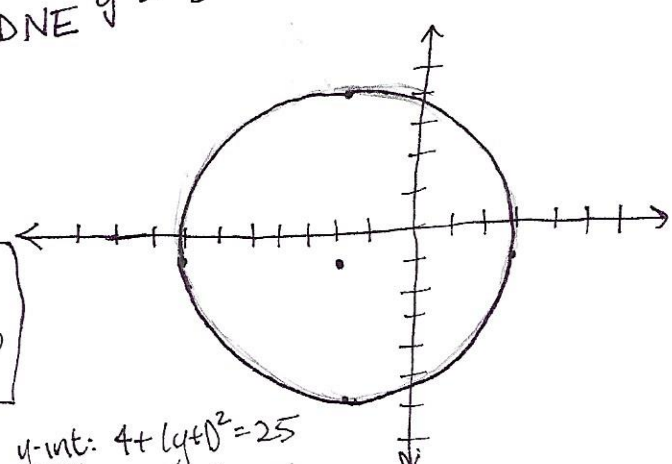
   $(x^2 + 4x) + (y^2 + 2y) = 20$

   $(x^2 + 4x + 4) + (y^2 + 2y + 1) = 20 + 4 + 1$

   $(x+2)^2 + (y+1)^2 = 25$

   Center: $(-2, -1)$
   Radius: 5

   Intercepts:
   $(-2 + 2\sqrt{6})$, $(-2 - 2\sqrt{6})$,
   $(-1 + \sqrt{21})$, $(-1 - \sqrt{21})$



   x-int: $(x+2)^2 + 1 = 25$

   $(x+2)^2 = 24$

   $x + 2 = \pm 2\sqrt{6} \Rightarrow x = -2 \pm 2\sqrt{6}$

   y-int: $4 + (y+1)^2 = 25$

   $(y+1)^2 = 21$

   $y + 1 = \pm\sqrt{21}$

   $y = -1 \pm \sqrt{21}$

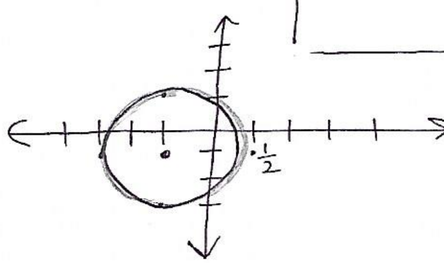
$\boxed{\text{Intercepts: } \left(-\frac{1}{2} \pm \frac{\sqrt{3}}{2}, 0\right) \quad \left(0, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}\right)}$

(c) $x^2 + y^2 + x + y - \frac{1}{2} = 0$

$(x^2+x)+(y^2+y) = \frac{1}{2}$

$(x^2+x+\frac{1}{4})+(y^2+y+\frac{1}{4}) = \frac{1}{2}+\frac{1}{2}$

$(x+\frac{1}{2})^2+(y+\frac{1}{2})^2 = 1$



$\boxed{\text{Center: } \left(-\frac{1}{2}, -\frac{1}{2}\right) \quad \text{Radius: } 1}$

x-ints: $(x+\frac{1}{2})^2+\frac{1}{4}=1$

$(x+\frac{1}{2})^2=\frac{3}{4}$

$x+\frac{1}{2} = \pm\frac{\sqrt{3}}{2} \Rightarrow x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$

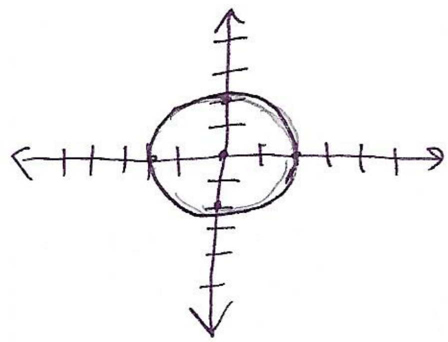
y-ints: $(y+\frac{1}{2})^2 = \frac{3}{4}$

$y = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$

(d) $3x^2 + 3y^2 - 12 = 0$

$x^2+y^2-4=0$

$x^2+y^2=4$



$\boxed{\text{Center: } (0,0) \quad \text{Radius: } 2}$ $\boxed{\text{Intercepts: } (2,0), (-2,0), (0,2), (0,-2)}$

x-ints: $x^2=4$, $x=\pm 2$ y-ints: $y^2=4$, $y=\pm 2$

3. Find the standard equation of each circle.

(a) Center (-3,1) and tangent to the y-axis.

$r=3$, $(h,k)=(-3,1)$

$\boxed{(x+3)^2+(y-1)^2=9}$

(b) With endpoints of a diameter at (4,3) and (0,1).

Center is the midpoint of the diameter

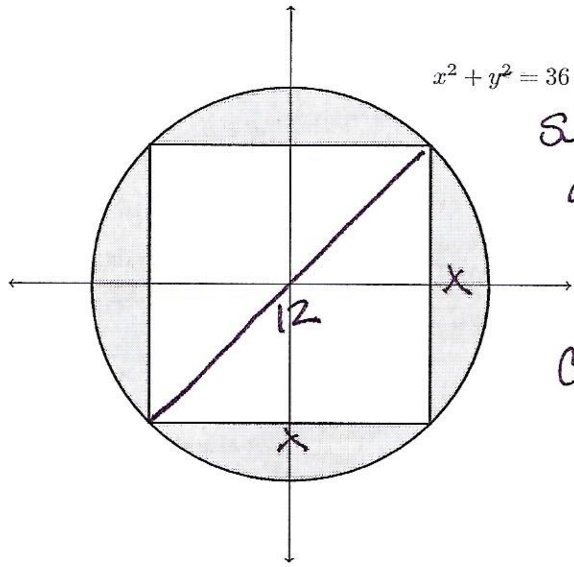
$(h,k) = \left(\frac{4+0}{2}, \frac{3+1}{2}\right) = (2,2)$

Radius is half the distance between the points.

$d = \sqrt{4^2+(3-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$r = \sqrt{5}$ $\therefore \boxed{(x-2)^2+(y-2)^2=5}$

4. Find the area of the shaded region, assuming that the quadrilateral inside is a square.



Since the radius is 6, the diagonal of the square is length 12. Further, since the sides of a square are all equal, call them x:

$$x^2 + x^2 = 144 \quad \text{(Pythagorean Thm)}$$

$$2x^2 = 144$$

$$x^2 = \boxed{72 = A_{sq.}}$$

$\boxed{A_{cir} = \pi r^2 = 36\pi}$

$\therefore$ the area of the shaded region is

$$A_{cir} - A_{sq} = \boxed{36\pi - 72 \ u^2}$$

5. The original Ferris wheel was built in 1893 by Pittsburgh, Pennsylvania, builder George W. Ferris. The Ferris wheel was originally built for the 1893 World's Fair in Chicago, but was also later reconstructed for the 1904 World's Fair in St. Louis. It had a maximum height of 264 feet and a wheel diameter of 250 feet. Find an equation for the wheel if the center is on the y-axis.

$d = 250 \Rightarrow r = 125$

Height $= 264 \Rightarrow$ Center $= 264 - 125 = 139$

Since we will place the wheel on the y-axis, the center will correspond to $(0, 139)$.

$\therefore$ an equation for the wheel will be

$$x^2 + (y - 139)^2 = 125^2$$

$$\boxed{x^2 + (y - 139)^2 = 15625}$$