Math 1120F - Fall 2014
Worksheet 14 - §2.5

Name(s): KEY

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Work with partners in groups of 2-4. **This is required.**

1. The monthly payment p on a mortgage varies directly with the amount borrowed B. If the monthly payment on a 15-year mortgage is \$8.99 for every \$1000 borrowed, find a linear equation that relates the monthly payment p to the amount borrowed B for a mortgage with the same terms. The find the monthly payment p when the amount borrowed B is \$175,000.

$p = kB$

$8.99 = k \cdot 1000$

$k = 0.00899$

$\therefore \boxed{p = 0.00899B}$

If $B = 175000$,

$p = 0.00899B$

$\boxed{p = 1573.25}$

2. The distance s that an object falls is directly proportional to the square of the time t of the fall. If an object falls 16 feet in 1 second, how far will it fall in 3 seconds? How long will it take the object to fall 64 feet?

$s = kt^2$

$16 = k(1)^2$

$\therefore s = 16t^2$

If $t = 3$:

$s = 16(3)^2$

$\boxed{s = 144 ft}$

If $s = 64$:

$64 = 16t^2$

$t^2 = 4$

$t = \pm 2$ discard neg

$\boxed{t = 2 sec}$

3. The velocity v of a falling object is directly proportional to the time t of the fall. If, after 2 seconds, the velocity of the object is 64 feet per second, what will its velocity be after 3 seconds?

$V = kt$

$64 = k(2)$

$k = 32$

$\therefore V = 32t$

If $t = 3$:

$V = 32(3)$

$\boxed{V = 96 ft/s}$

4. The rate of vibration of a string under constant tension varies inversely with the length of the string. If a string is 48 inches long and vibrates 256 times per second, what is the length of a string that vibrates 576 times per second?

$V = \frac{k}{L}$

$256 = \frac{k}{48}$

$k = 12288$

$\therefore V = \frac{12288}{L}$

If $V = 576$,

$576 = \frac{12288}{L}$

$L = \frac{12288}{576}$

$\boxed{L = \frac{64}{3} in.}$

5. The cost C of roasted almonds varies directly with the number A of pounds of almonds purchased. If the cost is \$23.75 when 5 pounds is purchased, find a linear equation that relates the cost C to the number of pounds of almonds purchased A. Then find the cost C when the number of pounds purchased is 3.5.

$C = kA$

$23.75 = k5$

$k = 4.75$

$\therefore \boxed{C = 4.75A}$

If $A = 3.5$:

$C = 4.75(3.5)$

$\boxed{C = \$16.63}$

6. The weight of an object above the surface of the Earth varies inversely with the square of the of the distance from the center of the Earth. If Maria weights 125 pounds when she is on the surface of the Earth (3960 miles from center), determine Maria's weight when she is at the top of Mount McKinley (3.8 miles from the surface of the Earth).

$W = \frac{k}{d^2}$

$125 = \frac{k}{(3960)^2}$

$k = 1960200000$

$\therefore W = \frac{1.9602E9}{d^2}$

If $d = 3.8 + 3960$:

$W = \frac{1.9602E9}{(3963.8)}$

$\boxed{W = 124.76 \text{ lbs}}$

7. The volume V of an ideal gas varies directly with temperature T and inversely with pressure P. Write an equation relating V, T, and P, using k as the constant of proportionality. If a cylinder contains oxygen at 300 K and a pressure of 15 atmospheres in a volume of 100 liters, what is the constant of proportionality k? If a piston is lowered into the cylinder, decreasing the volume occupied by the gas to 80 liters and raising the temperature to 310 K, what is the gas pressure?

$V = \frac{kT}{P}$

$100 = \frac{k(300)}{15}$

$\boxed{k = 5}$

$\therefore \boxed{V = \frac{5T}{P}}$

If $V = 80, T = 310$

$V = \frac{5T}{P}$

$80 = \frac{5(310)}{P}$

$P = \frac{5(310)}{80}$

$\boxed{P = 19.375 \text{ atm}}$

8. The electrical resistance of a wire varies directly with the length of the wire and inversely with the square of the diameter of the wire. If a wire 432 feet long and 4 millimeters wide has a resistance of 1.24 ohms, find the length of a wire of the same material whose resistance is 1.44 ohms and whose diameter is 3 millimeters.

$$R = \frac{kl}{d^2}$$

$$1.24 = \frac{k(432)}{16}$$

$$k = \frac{1.24}{27}$$

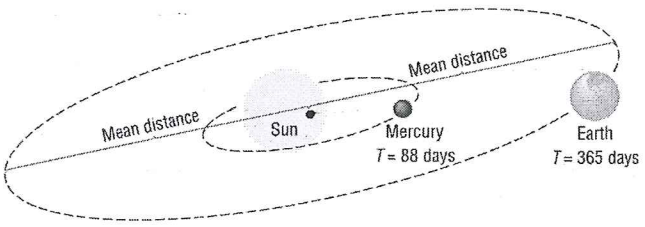
$$\therefore R = \frac{1.24l}{27d^2}$$

If $R = 1.44$, $d = 3$:

$$1.44 = \frac{1.24l}{27(9)}$$

$$\boxed{l = 282.2 \text{ ft}}$$

9. Kepler's Third Law of Planetary Motion states that the square of the period of revolution T of a planet varies directly with the cube of its mean distance a from the Sun. If the mean distance of Earth from the Sun is 93 million miles, what is the mean distance of the planet Mercury from the Sun, given that Mercury has a "year" of 88 days?



$$T^2 = ka^3$$

$$365^2 = k(93)^3$$

$$k = \frac{365^2}{93^3}$$

$$\therefore T^2 = \frac{365^2}{93^3} a^3$$

If $T = 88$ days:

$$88^2 = \frac{365^2}{93^3} a^3$$

$$a^3 = \frac{88^2 \cdot 93^3}{365^2}$$

$$a = \sqrt[3]{\frac{88^2 \cdot 93^3}{365^2}}$$

$$\boxed{a \doteq 36 \text{ million miles}}$$