Math 1120F - Fall 2014
Worksheet 15 - §R6, Factoring

Name(s): KEY

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Work with partners in groups of 2-4. **This is required.**

1. Using synthetic division, evaluate the following. Give your answer as one expression.

(a) $\dfrac{x^3+2x^2-3x+1}{x+1}$

$$\begin{array}{r|cccc} -1 & 1 & 2 & -3 & 1 \\ & \downarrow & -1 & -1 & 4 \\ \hline & 1 & 1 & -4 & 5 \end{array}$$

$$\therefore \frac{x^3+2x^2-3x+1}{x+1} = x^2+x-4+\frac{5}{x+1}$$

(b) $\dfrac{3x^3+2x^2-x+3}{x-3}$

$$\begin{array}{r|cccc} 3 & 3 & 2 & -1 & 3 \\ & \downarrow & 9 & 33 & 96 \\ \hline & 3 & 11 & 32 & 99 \end{array}$$

$$\therefore \frac{3x^3+2x^2-x+3}{x-3} = 3x^2+11x+32+\frac{99}{x-3}$$

(c) $\dfrac{-4x^3+2x^2-x+1}{x+2}$

$$\begin{array}{r|cccc} -2 & -4 & 2 & -1 & 1 \\ & \downarrow & 8 & -20 & 42 \\ \hline & -4 & 10 & -21 & 43 \end{array}$$

$$\therefore \frac{-4x^3+2x^2-x+1}{x+2} = -4x^2+10x-21+\frac{43}{x+2}$$

(d) $\dfrac{x^5-4x^3+x}{x+3}$

$$\begin{array}{r|cccccc} -3 & 1 & 0 & -4 & 0 & 1 & 0 \\ & \downarrow & -3 & 9 & -15 & 45 & -138 \\ \hline & 1 & -3 & 5 & -15 & 46 & -138 \end{array}$$

$$\therefore \frac{x^5-4x^3+x}{x+3} = x^4-3x^3+5x^2-15x+46+\frac{-138}{x+3}$$

(e) $\dfrac{4x^6-3x^4+x^2+5}{x-1}$

$$\begin{array}{r|ccccccc} 1 & 4 & 0 & -3 & 0 & 1 & 0 & 5 \\ & \downarrow & 4 & 4 & 1 & 1 & 2 & 2 \\ \hline & 4 & 4 & 1 & 1 & 2 & 2 & 7 \end{array}$$

$$\therefore \frac{4x^6-3x^4+x^2+5}{x-1} = 4x^5+4x^4+x^3+x^2+2x+2+\frac{7}{x-1}$$

2. Determine if the following polynomials have the given factor using synthetic division. If yes, what do you know to be a root of the polynomial?

(a) $-4x^3 + 5x^2 + 8; \quad x + 3$

$$\begin{array}{r|rrrr} -3 & -4 & 5 & 0 & 8 \\ & \downarrow & 12 & -51 & 153 \\ \hline & 4 & -17 & 51 & 161 \end{array}$$

Since the remainder is not zero, $x+3$ is not a factor.

(b) $3x^6 + 82x^3 + 27; \quad x + 3$

$$\begin{array}{r|rrrrrrr} -3 & 3 & 0 & 0 & 82 & 0 & 0 & 27 \\ & \downarrow & -9 & 27 & -81 & -3 & 9 & -27 \\ \hline & 3 & -9 & 27 & 1 & -3 & 9 & 0 \end{array}$$

Since the remainder is zero, $x+3$ is a factor and -3 is a root of the polynomial.

(c) $4x^6 - 64x^4 + x^2 - 15; \quad x + 4$

$$\begin{array}{r|rrrrrrr} -4 & 4 & 0 & -64 & 0 & 1 & 0 & -15 \\ & \downarrow & -16 & 64 & 0 & 0 & -4 & 16 \\ \hline & 4 & -16 & 0 & 0 & 1 & -4 & 1 \end{array}$$

Since the remainder is not zero, $x+4$ is not a factor.

(d) $2x^4 - x^3 + 2x - 1; \quad x - \frac{1}{2}$

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -1 & 0 & 2 & -1 \\ & \downarrow & 1 & 0 & 0 & 1 \\ \hline & 2 & 0 & 0 & 2 & 0 \end{array}$$

Since the remainder is zero, $x - \frac{1}{2}$ is a factor and $\frac{1}{2}$ is a root of the polynomial.

(e) $3x^4 + x^3 - 3x + 1; \quad 3x + 1$

$3x+1 = x + \frac{1}{3}$

$$\begin{array}{r|rrrrr} -\frac{1}{3} & 3 & 1 & 0 & -3 & 1 \\ & \downarrow & -1 & 0 & 0 & 1 \\ \hline & 3 & 0 & 0 & -3 & 2 \end{array}$$

Since the remainder is not zero, $x + \frac{1}{3}$ is not a factor.

3. Write the following rational expressions in lowest terms. Be sure to include domain restrictions.

(a) $\frac{15x^2+24x}{3x^2}$ $D=\{x \mid x\neq 0\}$

$$\frac{15x^2+24x}{3x^2} = \frac{3x(5x+8)}{3x^2} = \frac{5x+8}{x}, \quad x\neq 0$$

(b) $\frac{2x^2+5x-3}{1-2x}$ $D=\{x \mid x\neq \frac{1}{2}\}$

$$\frac{2x^2+5x-3}{1-2x} = \frac{(2x-1)(x+3)}{-(2x-1)} = \frac{x+3}{-1} = -x-3, \quad x\neq \frac{1}{2}$$

4. Factor the following completely.

(a) $2(3x+4)^2+(2x+3)\cdot 2(3x+4)\cdot 3$

$$= 2(3x+4)\left((3x+4)+(2x+3)\cdot 3\right)$$
$$= 2(3x+4)(3x+4+6x+9)$$
$$= \boxed{2(3x+4)(9x+13)}$$

(b) $2x(2x+5)+x^2\cdot 2$

$$= 2x\left((2x+5)+x\right)$$
$$= \boxed{2x(3x+5)}$$

(c) $3x^2(3x+4)^2+x^3\cdot 2(3x+4)\cdot 3$

$$= 3x^2(3x+4)\left((3x+4)+2x\right)$$
$$= \boxed{3x^2(3x+4)(5x+4)}$$

(d) $4(x+5)^3(x-1)^2+(x+5)^4\cdot 2(x-1)$

$$= 2(x+5)^3(x-1)\left(2(x-1)+(x+5)\right)$$
$$= 2(x+5)^3(x-1)(2x-2+x+5)$$
$$= 2(x+5)^3(x-1)(3x+3)$$
$$= \boxed{6(x+5)^3(x-1)(x+1)}$$