Math 1120F - Fall 2014
Worksheet 16 - §5.1, 5.5

Name(s): KEY

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Work with partners in groups of 2-4. **This is required.**

1. For the following, state which are functions. For those that are, give the degree; for those that are not, say why not.
   (a) $g(x) = \frac{1-x^2}{2}$ polynomial function; degree 2
   (b) $h(x) = 1 - \frac{1}{x}$ not a polynomial because $\frac{1}{x} = x^{-1}$ (neg. exp)
   (c) $f(x) = 5x^4 - \pi x^3$ polynomial function; degree 4
   (d) $m(x) = \frac{x^2-5}{x^3}$ not a polynomial because x^{-3} (neg. exp)

2. For the following functions: (*a*) list zeros with their multiplicities; (*b*) determine behavior near zeros; (*c*) determine if the function crosses or touches the axis at each zero; (*d*) determine end behavior.
   (a) $F(x) = 4(x+4)(x+3)^3$

   zeros: -4, mult 1 $\Rightarrow$ cross
   -3, mult 3 $\Rightarrow$ cross

   End behavior: x^4

   Behavior:
   $x=-4$: $F(x) \doteq 4(x+4)(-4+3)^3$
   $= -4(x+4)$, line neg slope

   $x=3$: $\doteq 4(3+4)(x+3)^3$
   $F(x) = 28(x+3)^3$, cubic

   (b) $G(x) = -2(x^2+3)^3$

   zeros: $x^2+3=0$
   $x^2=-3$
   no real solution
   $\therefore$ graph does not cross x-axis

   End behavior: $-x^6$

   (c) $H(x) = 4x(x^2-3)$

   zeros: 0, mult 1 $\Rightarrow$ cross
   $\sqrt{3}$, mult 1 $\Rightarrow$ cross
   $-\sqrt{3}$, mult 1 $\Rightarrow$ cross

   End behavior: x^3

   $x=\sqrt{3}$: $H(x) \doteq 4\sqrt{3}(x^2-3)$, parabola opens up

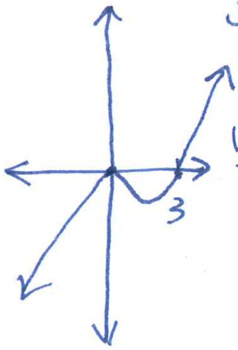
   $x=-\sqrt{3}$: $H(x) \doteq -4\sqrt{3}(x^2-3)$, parabola opens down

   Behavior:
   $x=0$: $H(x) \doteq 4x(0-3)$
   $= -12x$, line neg. slope

3. Analyze the following functions using the steps given in class. Sketch a rough graph of the function.

(a) $f(x) = x^2 (x-3)$

zeros: 0, mult 2 $\Rightarrow$ touch
3, mult 1 $\Rightarrow$ cross



intercepts:
x-ints: (0,0), (3,0)
y-int: (0,0)

End behavior: $x^3 \Rightarrow f \to -\infty$ as $x \to -\infty$
$f \to \infty$ as $x \to \infty$

0 3
$(-\infty,0)$ (0,3) $(3,\infty)$
-1 1 4
$\oplus\ominus$ $\oplus\ominus$ $\oplus\oplus$
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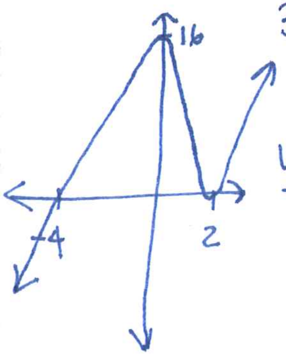
(b) $g(x) = (x+4)(x-2)^2$

zeros: -4, mult 1 $\Rightarrow$ cross
2, mult 2 $\Rightarrow$ touch



intercepts:
x-ints: (-4,0), (2,0)
y-int: (0,16)

End behavior: $x^3 \Rightarrow f \to -\infty$ as $x \to -\infty$
$f \to \infty$ as $x \to \infty$

-4 2
$(-\infty,-4)$ (-4,2) $(2,\infty)$
-5 0 3
$\ominus\oplus$ $\oplus$ $\oplus\oplus$
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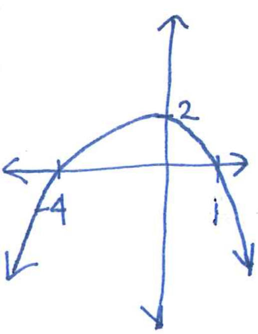
(c) $h(x) = -\frac{1}{2}(x+4)(x-1)^3$

zeros: -4, mult 1 $\Rightarrow$ cross
1, mult 3 $\Rightarrow$ cross



intercepts:
x-ints: (-4,0), (1,0)
y-int: (0,2)

End behavior: $-x^4 \Rightarrow f \to -\infty$ as $x \to -\infty$
$f \to -\infty$ as $x \to \infty$

-4 1
$(-\infty,-4)$ (-4,1) $(1,\infty)$
-5 0 2
$\ominus\ominus\ominus$ $\oplus$ $\ominus\oplus\oplus$
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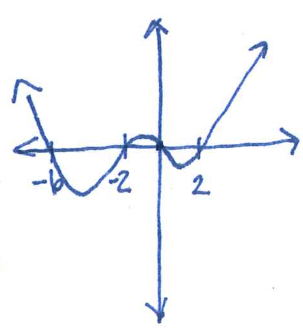
(d) $F(x) = 2x^4 + 12x^3 - 8x^2 - 48x$

factor by grouping:
$F(x) = (2x^2+12x)(x^2-4) = 2x(x+6)(x-2)(x+2)$

zeros: 0, mult 1 $\Rightarrow$ cross
-2, mult 1 $\Rightarrow$ cross
2, mult 1 $\Rightarrow$ cross
-6, mult 1 $\Rightarrow$ cross

End behavior: $x^4 \Rightarrow f \to \infty$ as $|x| \to \infty$

-6 -2 0 2
$(-\infty,-6)$ (-6,-2) (-2,0) (0,2) $(2,\infty)$
-10 -3 -1 1 3
$\ominus\ominus\ominus\ominus$ $\ominus\oplus\ominus\ominus$ $\ominus\oplus\ominus\oplus$ $\oplus\oplus\ominus\oplus$ $\oplus\oplus\oplus\oplus$
$\oplus$ $\ominus$ $\oplus$ $\ominus$ $\oplus$



intercepts:
x-ints: (0,0), (-2,0), (2,0), (-6,0)
y-int: (0,0)

4. Construct a polynomial with the following characteristics:

(a) zeros: -3, -1, 2, 5; degree 4

$f(x) = a(x+3)(x+1)(x-2)(x-5)$ Let $a=1$. $\boxed{f(x) = (x+3)(x+1)(x-2)(x-5)}$

(b) zeros: -2, multiplicity 2; 4, multiplicity 1; degree 3

$g(x) = a(x+2)^2(x-4)$. Let $a=1$. $\boxed{g(x) = (x+2)^2(x-4)}$

5. Give the maximum number of zeros possible for the following polynomials. Do not attempt to find the zeros.

(a) $f(x) = -3x^7 + 2x^2 + 2$ degree is 7 so max number of zeros is 7

(b) $g(x) = x^5 + 1$ degree is 5 so max number of zeros is 5

6. Use the Intermediate Value Theorem to show if the following functions have a zero in the given interval. Clearly explain your answer.

(a) $H(x) = x^4 + 8x^3 - x^2 + 2$; $[-5, -4]$

$H(-5) = (-5)^4 + 8(-5)^3 - (-5)^2 + 2 < 0$

$H(-4) = (-4)^4 + 8(-4)^3 - (-4)^2 + 2 = 4^4 + 8(4)^3 - 4^2 + 2 > 0$

Since $H(-5)$ and $H(-4)$ differ by a sign and $H(x)$ is continuous, there is at least one zero in $[-5, -4]$.

(b) $G(x) = 3x^3 - 10x + 9$; $[0, 1]$

$G(0) = 9 > 0$

$G(1) = 3 - 10 + 9 > 0$

Since $G(0)$, $G(1)$ do not differ by a sign, we cannot say if there is a zero in $[0,1]$.

7. Use the Remainder Theorem to find the remainder when the function $f(x)$ is divided by $x - c$.

(a) $3x^4 - 6x^3 - 5x + 10$; $x - 2$

$3(2)^4 - 6(2)^3 - 5(2) + 10 = 3(16) - 6(8) - 10 + 10 = 48 - 48 = 0$

$\therefore$ the remainder is zero.

(b) $2x^4 - x^3 + 2x - 1$; $x - \frac{1}{2}$

$2(\frac{1}{2})^4 - (\frac{1}{2})^3 + 2(\frac{1}{2}) - 1 = 2(\frac{1}{16}) - \frac{1}{8} + 1 - 1 = 0$

$\therefore$ the remainder is zero.

8. Use the Rational Root Theorem to factor the following polynomials over the real numbers ($\mathbb{R}$).

(a) $p(x) = x^3 + 8x^2 + 11x - 20$

$p = \pm 1, \pm 2, \pm 5, \pm 4, \pm 10, \pm 20 \Rightarrow \frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$q = \pm 1$

$$\begin{array}{r|rrrr} 1 & 1 & 8 & 11 & -20 \\ & \downarrow & 1 & 9 & 20 \\ \hline & 1 & 9 & 20 & 0 \end{array}$$

$\leftarrow$ depressed equation

$\therefore p(x) = (x^2 + 9x + 20)(x-1)$

$\boxed{p(x) = (x+5)(x+4)(x-1)}$

(b) $q(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$

$p = \pm 1, \pm 3 \Rightarrow \frac{p}{q} = \pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}$

$q = \pm 1, \pm 2$

$$\begin{array}{r|rrrrr} -1 & 2 & 1 & -7 & -3 & 3 \\ & \downarrow & -2 & 1 & 6 & -3 \\ \hline & 2 & -1 & -6 & 3 & 0 \end{array}$$

$\therefore q(x) = (x+1)(2x^3 - x^2 - 6x + 3)$

Factor the depressed equation

Depressed equation:

$2x^3 - x^2 - 6x + 3$

$= 2x(x^2 - 3) - (x^2 - 3)$

$= (2x-1)(x^2-3)$

$\boxed{\therefore q(x) = (x+1)(2x-1)(x^2-3)}$

(c) $r(x) = x^4 + x^3 - 3x^2 - x + 2$

$p = \pm 1, \pm 2 \Rightarrow \frac{p}{q} = \pm 1, \pm 2$

$q = \pm 1$

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & -3 & -1 & 2 \\ & \downarrow & 1 & 2 & -1 & -2 \\ \hline & 1 & 2 & -1 & -2 & 0 \end{array}$$

$\therefore r(x) = (x-1)(x^3 + 2x^2 - x - 2)$

Factor again using RRT:

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -1 & -2 \\ & \downarrow & 1 & 3 & 2 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$\therefore r(x) = (x-1)(x-1)(x^2 + 3x + 2)$

Depressed equation:

$x^2 + 3x + 2 = (x+1)(x+2)$

$\boxed{\therefore r(x) = (x-1)^2(x+1)(x+2)}$