Math 1120F - Fall 2014
Worksheet 17 - §5.6

Name(s): KEY

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Work with partners in groups of 2-4. **This is required.**

1. Information is given about a polynomial whose coefficients are real numbers. Find the remaining zeros of the polynomial.

   (a) Degree 4; zeros: $2 - i$, i

   $2+i$

   $-i$

   (b) Degree 6; zeros: 2, $2 + i$, $-3 - i$, 0

   $2-i$

   $-3+i$

2. Form a polynomial with real coefficients having the following characteristics.

   (a) Degree 4; zeros: i; $1 + 2i$

   $f(x) = a(x-i)(x-(1+2i))(x+i)(x-(1-2i))$

   $= a(x^2+1)(x^2-2x+5)$

   Let $a=1$.

   $\boxed{f(x) = x^4 - 2x^3 + 6x^2 - 2x + 5}$

   (b) Degree 5; zeros: 2; $-2i$; $1 + i$

   $g(x) = a(x-2)(x-2i)(x+2i)(x-(1+i))(x-(1-i))$

   $= a(x-2)(x^2+4)(x^2-2x+2)$

   Let $a=1$.

   $\boxed{g(x) = x^5 - 4x^4 + 10x^3 - 20x^2 + 24x - 16}$

3. Use the given zero to find all remaining zeros.

(a) $f(x) = 2x^4 + 5x^3 + 5x^2 + 20x - 12$; zero: $2i$ $(x-2i)(x+2i) = (x^2+4)$ is a factor

$$x^2+4 \overline{)\, 2x^4+5x^3+5x^2+20x-12}$$

$2x^2+5x-3$

$-(2x^4 \quad +8x^2)$

$5x^3-3x^2+20x$

$-(5x^3 \quad +20x)$

$-3x^2 \quad -12$

$-(-3x^2 \quad -12)$

0

$\therefore$ the remaining zeros are $-2i$, $\frac{1}{2}$, -3

$2x^2+5x-3 = (2x-1)(x+3)$

(b) $g(x) = x^4 - 7x^3 + 14x^2 - 38x - 60$; zero: $1+3i$ $(x-(1+3i))(x-(1-3i)) = x^2-2x+10$ is a factor

$$x^2-2x+10 \overline{)\, x^4-7x^3+14x^2-38x-60}$$

x^2-5x-6

$-(x^4-2x^3+10x^2)$

$-5x^3+4x^2-38x$

$-(-5x^3+10x^2-50x)$

$-6x^2+12x-60$

$-(-6x^2+12x-60)$

0

$\therefore$ the remaining zeros are $1-3i$, -1, 6

$x^2-5x-6 = (x+1)(x-6)$

(c) $h(x) = x^3 + 3x^2 + 25x + 75$; zero: $-5i$ $(x-5i)(x+5i) = x^2+25$ is a factor

$$x^2+25 \overline{)\, x^3+3x^2+25x+75}$$

$x+3$

$-(x^3 \quad +25x)$

$3x^2 \quad +75$

$-(3x^2 \quad +75)$

0

$\therefore$ the remaining zeros are $5i$, -3

4. Find all zeros of the following functions. Write the function in factored form.

(a) $f(x) = x^3 - 1$

$x=1$ is one root

$$1 \,|\, 1 \quad 0 \quad 0 \quad -1$$
$$\downarrow \quad 1 \quad 1 \quad 1$$
$$1 \quad 1 \quad 1 \quad 0$$

$\therefore x^3-1 = (x-1)(x^2+x+1)$

for depressed eqn:

$x = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

zeros: $1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$f(x) = (x-1)(x+\frac{1}{2}-\frac{\sqrt{3}}{2}i)(x+\frac{1}{2}+\frac{\sqrt{3}}{2}i)$

(b) $g(x) = x^3 - 8x^2 + 25x - 26$

by RRT, possible zeros are $\pm 1, \pm 2, \pm 13, \pm 26$

$$2 \,|\, 1 \quad -8 \quad 25 \quad -26$$
$$\downarrow \quad 2 \quad -12 \quad 26$$
$$1 \quad -6 \quad 13 \quad 0$$

Depressed eqn:

$x^2 - 6x + 13$

$x = \frac{+6 \pm \sqrt{36-4(13)}}{2} = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2}$

$= 3 \pm 2i$

zeros: $2, 3+2i, 3-2i$

$g(x) = (x-2)(x-3+2i)(x-3-2i)$

5. Find all solutions to the following equations.

(a) $x^4 = 1$

$x^4 - 1 = 0$

$(x^2-1)(x^2+1) = 0$

$x^2-1=0$ or $x^2+1=0$

$x^2=1$ $\quad$ $x^2=-1$

$x=\pm 1$ $\quad$ $x=\pm i$

soln: $1, -1, i, -i$

(b) $x^4 + 5x^2 = -4$

$x^4 + 5x^2 + 4 = 0$

Let $u = x^2$.

$u^2 + 5u + 4 = 0$

$(u+1)(u+4) = 0$

$u+1=0$ or $u+4=0$

$u=-1$ $\quad$ $u=-4$

$x^2=-1$ $\quad$ $x^2=-4$

$x=\pm i$ $\quad$ $x=\pm 2i$

soln: $i, -i, 2i, -2i$