Math 1120F - Fall 2014
Worksheet 18 - §5.2, 5.4

Name(s): KEY

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Work with partners in groups of 2-4. **This is required.**

1. Find the domain and any asymptotes of the following rational functions.

(a) $\frac{x^3-8}{x^2-5x+6} = \frac{(x-2)(x^2+2x+4)}{(x-2)(x-3)} = \frac{x^2+2x+4}{x-3}$, $x \neq 2, 3$

$\deg(p) = \deg(q)+1$ ∴ there is an oblique asymptote

$$x-3 \overline{)\, x^2+2x+4}$$

$x+5$

$-(x^2-3x)$

$5x+4$

$-(5x-15)$

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∴ the oblique asymptote is $y = x+5$

vertical asymptote is $x = 2,\ x = 3$

$D = \{x \mid x \neq 2, 3\}$

(b) $\frac{x^4-1}{x^2-x} = \frac{(x^2+1)(x^2-1)}{x(x-1)} = \frac{(x^2+1)(x-1)(x+1)}{x(x-1)} = \frac{(x^2+1)(x+1)}{x}$, $x \neq 0, 1$

$\deg(p) \geq \deg(q)+2$ there is no horizontal or oblique asymptote.

∴ the vertical asymptotes are $x = 0,\ x = 1$

$D = \{x \mid x \neq 0, 1\}$

2. Solve the following inequalities.

(a) $\frac{1}{x-2} < \frac{2}{3x-9}$

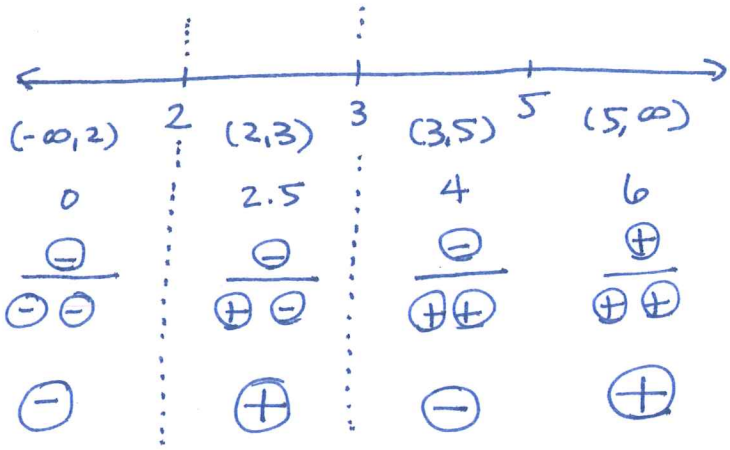
$\frac{1}{x-2} - \frac{2}{3x-9} < 0$

$\frac{3x-9-2(x-2)}{(x-2)(3x-9)} < 0$

$\frac{x-5}{(x-2)(3x-9)} < 0$

zeros: 5

asymp: 2, 3



Soln: $(-\infty, 2) \cup (3, 5)$

(b) $\frac{x-4}{2x+4} \geq 1$

$\frac{x-4}{2x+4} - 1 \geq 0$

$\frac{x-4}{2x+4} - \frac{2x+4}{2x+4} \geq 0$

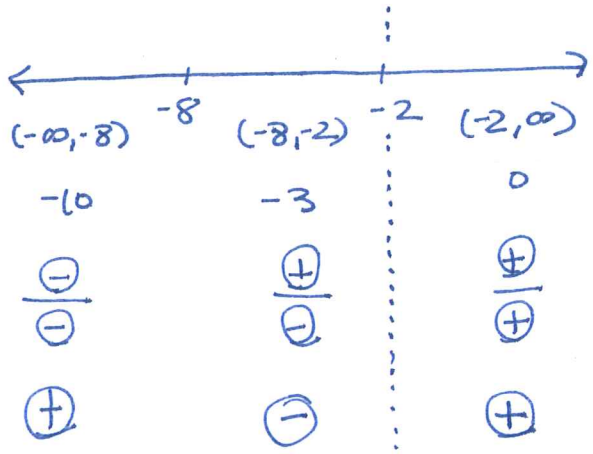
$\frac{x-4-2x-4}{2x+4} \geq 0$

$\frac{-x-8}{2x+4} \geq 0$

$\frac{x+8}{2x+4} \leq 0$

zeros: -8

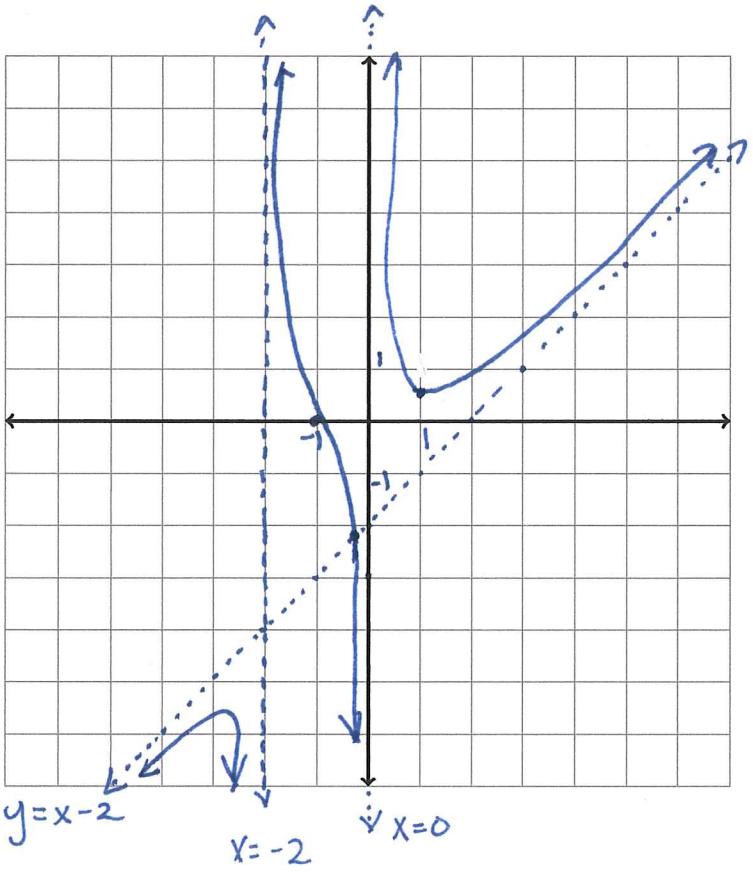
asymp: -2



Soln: $[-8, -2)$

3. Graph the following rational function. Clearly label any asymptotes and intercepts.

$$R(x) = \frac{x^3+1}{x^2+2x} = \frac{(x+1)(x^2-x+1)}{x(x+2)}$$



$D = \{x \mid x \neq 0, -2\}$

intercepts: $(-1, 0)$

asymptotes: $x = -2, x = 0$

Since $\deg(p) = \deg(q)+1$, there is an oblique asymptote.

$$\begin{array}{r} x-2 \\ x^2+2x \,\overline{)\, x^3+0x^2+0x+1} \\ -(x^3+2x^2) \\ \hline -2x^2+0x \\ -(-2x^2-4x) \\ \hline 4x+1 \end{array}$$

∴ the oblique asymptote is $y = x-2$.

Check for asymptote crossing:

$$\frac{x^3+1}{x^2+2x} = x-2$$

$$x^3+1 = x^3-4x$$

$$1 = -4x$$

$$x = -\frac{1}{4}$$

∴ graph crosses at $x = -\frac{1}{4}$