

Work with partners in groups of 2-4. This is required.

1. Suppose  $f(x) = \frac{x}{x+3}$  and  $g(x) = \frac{2}{x}$ . Find  $(f \circ g)(x)$  and its domain.

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2}{x}\right) = \frac{\frac{2}{x}}{\frac{2}{x} + 3}$$

$$= \frac{\frac{2}{x}}{\frac{2+3x}{x}} = \boxed{\frac{2}{2+3x} = (f \circ g)(x)}$$

$$D_f: \{x \mid x \neq -3\}$$

$$D_g: \{x \mid x \neq 0\}$$

when does  $g(x) = -3$ ?

$$\frac{2}{x} = -3$$

$$2 = -3x \Rightarrow x = -\frac{2}{3}$$

$\therefore$  the domain of the composition function is given by domain of the "inside" function and when "inside" function "breaks" "outside" function:  $\boxed{\{x \mid x \neq 0, x \neq -\frac{2}{3}\}}$

2. Suppose  $H(x) = (2x+3)^4$ . Find  $f(x)$  and  $g(x)$  such that  $(f \circ g)(x) = H(x)$ .

$$f(x) = x^4$$

$$g(x) = 2x+3$$

$$\Rightarrow (f \circ g)(x) = (2x+3)^4$$

\* A different option:

$$f(x) = (x+3)^4$$

$$g(x) = 2x.$$

3. Suppose  $f(x) = \frac{2x+3}{x+2}$ . Find the inverse function of  $f(x)$ . State its domain and range.

$$y = \frac{2x+3}{x+2}$$

$$y(x+2) = 2x+3$$

$$yx+2y = 2x+3$$

$$yx-2x = 3-2y$$

$$x(y-2) = 3-2y$$

$$x = \frac{3-2y}{y-2}$$

$$D_f = R_{f^{-1}}$$

$$R_f = D_{f^{-1}}$$

$$D_{f^{-1}} = \{x \mid x \neq 2\}$$

$$R_{f^{-1}} = \{x \mid x \neq -2\}$$

4. Suppose  $F(x) = \sqrt{x+1}$  and  $G(x) = 3x$ . Find the following values:

(a)  $(F \circ F)(1)$ :  $\sqrt{\sqrt{2}+1}$

(b)  $(G \circ F)(2)$ :  $3\sqrt{3}$ .

$$a) F(F(1)) = F(\sqrt{1+1}) = F(\sqrt{2}) = \sqrt{\sqrt{2}+1}$$

$$b) G(F(2)) = G(\sqrt{2+1}) = G(\sqrt{3}) = 3\sqrt{3}.$$