Math 1120F - Fall 2014
Worksheet 20 - §13.1, 13.5

Name(s): KEY

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Work with partners in groups of 2-4. **This is required.**

1. Express $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{13}{14}$ in summation notation. Write out the first few terms to check your work.

$$\sum_{k=1}^{13} \frac{k}{k+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots$$

2. Evaluate the following sums.

(a) $\sum_{k=1}^{8} (k^2 + 4)$

$$\sum_{k=1}^{8} k^2 + \sum_{k=1}^{8} 4 = \frac{8(8+1)(2(8)+1)}{6} + 8(4) = \frac{8(9)(17)}{6} + 32 = \boxed{236}$$

(b) $\sum_{k=6}^{40} (-3k)$

$$\sum_{k=6}^{40}(-3k) = \sum_{k=1}^{40}(-3k) - \sum_{k=1}^{5}(-3k) = -3\sum_{k=1}^{40} k + 3\sum_{k=1}^{5} k = \frac{-3(40)(40+1)}{2} + \frac{3(5)(5+1)}{2}$$

$$= -2460 + 45 = \boxed{-2415}$$

3. Evaluate $\binom{37}{29}$ and reduce as much as possible without a calculator. Give your answer as product; do *not* try to multiply it out.

$$\binom{37}{29} = \frac{37!}{29!(37-29)!} = \frac{37!}{29!\,8!} = \frac{37\cdot 36\cdot 35\cdot 34\cdot 33\cdot 32\cdot 31\cdot 30}{8!} = \frac{37\cdot 36\cdot 35\cdot 34\cdot 33\cdot 30}{7\cdot 6\cdot 5\cdot 3\cdot 2}$$

$$= \boxed{37\cdot 34\cdot 31\cdot 11\cdot 6\cdot 15}$$

4. Use the Binomial Theorem to expand $(x-6)^6$.

$$\sum_{k=0}^{6} \binom{6}{k} x^{6-k}(-6)^k = \binom{6}{0}x^6(-6)^0 + \binom{6}{1}x^5(-6)^1 + \binom{6}{2}x^4(-6)^2$$
$$+ \binom{6}{3}x^3(-6)^3 + \binom{6}{4}x^2(-6)^4 + \binom{6}{5}x^1(-6)^5 + \binom{6}{6}x^0(-6)^6$$

$$= x^6 + 6(-6)x^5 + 15(36)x^4 + 20(-216)x^3 + 15(1296)x^2 + 6(-7776)x + 46656$$

$$= \boxed{x^6 - 36x^5 + 540x^4 - 4320x^3 + 19440x^2 - 46656x + 46656}$$

5. Find the coefficient of x^2 in the expansion of $(3x-2)^8$.

We require $8-k=2$ term.

$\boxed{k=6}$

$$\binom{8}{6}(3x)^{8-6}(-2)^6 = 28(9x^2)(64) = 16128x^2$$

$\boxed{\therefore \text{ the coefficient is } 16128}$