Math 1120F - Fall 2014
Worksheet 20 - §13.1, 13.5

Name(s): KEY

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Work with partners in groups of 2-4. **This is required.** Many different ways to solve these!

1. Solve the system $\begin{cases} 3x+3y=3 & (1) \\ 4x+2y=\frac{8}{3} & (2) \end{cases}$

$3x+3y=3$

$x+y=1$

$y=1-x. \quad (1)$

Sub (1) into (2):

$4x+2(1-x)=\frac{8}{3}$

$4x+2-2x=\frac{8}{3}$

$2x=\frac{8}{3}-\frac{6}{3}$

$\Rightarrow 2x=\frac{2}{3}$

$x=\frac{1}{3}$

Sub into (1):

$3(\frac{1}{3})+3y=3$

$3y=2$

$y=\frac{2}{3}$

$\boxed{\text{Sol}^{\underline{n}}: x=\frac{1}{3}, y=\frac{2}{3}}$

2. Solve the system $\begin{cases} x+y-z=6 \\ 2x-2y+z=-5 \\ x+3y-2z=14 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 2 & -2 & 1 & -5 \\ 1 & 3 & -2 & 14 \end{array}\right] \begin{array}{l} R_2=-2r_1+r_2 \\ R_3=-r_1+r_2 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -4 & 3 & -17 \\ 0 & 2 & -1 & 8 \end{array}\right] R_2=-\frac{1}{4}r_2 \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & 1 & -\frac{3}{4} & +\frac{17}{4} \\ 0 & 2 & -1 & 8 \end{array}\right] \begin{array}{l} R_1=-r_2+r_1 \\ R_3=-2r_2+r_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{4} & \frac{7}{4} \\ 0 & 1 & -\frac{3}{4} & \frac{17}{4} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \end{array}\right] R_3=2r_3 \sim \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{4} & \frac{7}{4} \\ 0 & 1 & -\frac{3}{4} & \frac{17}{4} \\ 0 & 0 & 1 & -1 \end{array}\right] \begin{array}{l} R_1=\frac{1}{4}r_3+r_1 \\ R_2=\frac{3}{4}r_3+r_2 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{3}{2} \\ 0 & 1 & 0 & \frac{7}{2} \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$\boxed{\therefore x=\frac{3}{2}, y=\frac{7}{2}, z=-1}$

3. Solve the system $\begin{cases} 3x - 2y + 2z = 6 \\ 7x - 3y + 2x = -1 \\ 2x - 3y + 4z = 0 \end{cases}$

$$\left[\begin{array}{ccc|c} 3 & -2 & 2 & 6 \\ 7 & -3 & 2 & -1 \\ 2 & -3 & 4 & 0 \end{array}\right] \xrightarrow{R_1 = \frac{1}{3}r_1} \sim \left[\begin{array}{ccc|c} 1 & -\frac{2}{3} & \frac{2}{3} & 2 \\ 7 & -3 & 2 & -1 \\ 2 & -3 & 4 & 0 \end{array}\right] \begin{array}{l} \\ R_2 = -7r_1 + r_2 \\ R_3 = -2r_1 + r_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{2}{3} & \frac{2}{3} & 2 \\ 0 & \frac{5}{3} & -\frac{8}{3} & -15 \\ 0 & -\frac{5}{3} & \frac{8}{3} & -4 \end{array}\right] R_3 = r_2 + r_3 \quad \sim \left[\begin{array}{ccc|c} 1 & -\frac{2}{3} & \frac{2}{3} & 2 \\ 0 & \frac{5}{3} & -\frac{8}{3} & -15 \\ 0 & 0 & 0 & -19 \end{array}\right]$$

$\therefore$ the system is inconsistent and there is no solution

4. Solve using Cramer's Rule: $\begin{cases} 2x - 3y = -1 \\ 10x + 10y = 5 \end{cases}$

$$D = \begin{vmatrix} 2 & -3 \\ 10 & 10 \end{vmatrix} = 20 - (-30) = 50$$

$$Dx = \begin{vmatrix} -1 & -3 \\ 5 & 10 \end{vmatrix} = -10 - (-15) = 5$$

$$Dy = \begin{vmatrix} 2 & -1 \\ 10 & 5 \end{vmatrix} = 10 - (-10) = 20$$

$$\therefore\ x = \frac{Dx}{D} = \frac{5}{50} = \frac{1}{10}$$

$$y = \frac{Dy}{D} = \frac{20}{50} = \frac{2}{5}$$

5. Solve using Cramer's Rule: $\begin{cases} x+y-z=6 \\ 3x-2y+z=-5 \\ x+3y-2z=14 \end{cases}$

$$D=\begin{vmatrix} 1 & 1 & -1 \\ 3 & -2 & 1 \\ 1 & 3 & -2 \end{vmatrix} = 1\begin{vmatrix} -2 & 1 \\ 3 & -2 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} + (-1)\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix}$$

$$=(4-4)-(-6-1)-(9+2)=-3$$

$$Dx=\begin{vmatrix} 6 & 1 & -1 \\ -5 & -2 & 1 \\ 14 & 3 & -2 \end{vmatrix} = 6\begin{vmatrix} -2 & 1 \\ 3 & -2 \end{vmatrix} - 1\begin{vmatrix} -5 & 1 \\ 14 & -2 \end{vmatrix} + (-1)\begin{vmatrix} -5 & -2 \\ 14 & 3 \end{vmatrix}$$

$$=6(4-3)-1(10-14)-1(-15+28)=-3$$

$$Dy=\begin{vmatrix} 1 & 6 & -1 \\ 3 & -5 & 1 \\ 1 & 14 & -2 \end{vmatrix} = 1\begin{vmatrix} -5 & 1 \\ 14 & -2 \end{vmatrix} - 6\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} + (-1)\begin{vmatrix} 3 & -5 \\ 1 & 14 \end{vmatrix}$$

$$=1(10-14)-6(-6-1)-1(42+5)=-9$$

$$Dz=\begin{vmatrix} 1 & 1 & 6 \\ 3 & -2 & -5 \\ 1 & 3 & 14 \end{vmatrix} = 1\begin{vmatrix} -2 & -5 \\ 3 & 14 \end{vmatrix} - 1\begin{vmatrix} 3 & -5 \\ 1 & 14 \end{vmatrix} + 6\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix}$$

$$=1(-28+15)-1(42+5)+6(9+2)=6$$

$$\boxed{\therefore X=1,\ y=3,\ Z=-2}$$

6. Find the inverse of the matrix $\begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}$.

$$\left[\begin{array}{cc|cc} 6 & 5 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{array}\right] R_1=\tfrac{1}{6}r_1 \sim \left[\begin{array}{cc|cc} 1 & \frac{5}{6} & \frac{1}{6} & 0 \\ 2 & 2 & 0 & 1 \end{array}\right] R_2=-2r_1+r_2$$

$$\sim \left[\begin{array}{cc|cc} 1 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & \frac{1}{3} & -\frac{1}{3} & 1 \end{array}\right] R_2=3r_2 \sim \left[\begin{array}{cc|cc} 1 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 1 & -1 & 3 \end{array}\right] R_1=-\tfrac{5}{6}r_2+r_1$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & 1 & -\frac{5}{2} \\ 0 & 1 & -1 & 3 \end{array}\right]$$

$$\boxed{\therefore A^{-1}=\begin{bmatrix} 1 & -\frac{5}{2} \\ -1 & 3 \end{bmatrix}}$$

7. Solve the system using inverses: $\begin{cases} -4x + y = 0 \\ 6x - 2y = 14 \end{cases}$

Solve $A\underline{X} = B$ using $\underline{X} = A^{-1}B$

$A = \begin{bmatrix} -4 & 1 \\ 6 & -2 \end{bmatrix}$, $\underline{X} = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 14 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} -4 & 1 & 1 & 0 \\ 6 & -2 & 0 & 1 \end{array}\right] R_1 = -\tfrac{1}{4}r_1 \sim \left[\begin{array}{cc|cc} 1 & -\frac{1}{4} & -\frac{1}{4} & 0 \\ 6 & -2 & 0 & 1 \end{array}\right] R_2 = -6r_1 + r_2$$

$$\sim \left[\begin{array}{cc|cc} 1 & -\frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & -\frac{1}{2} & \frac{3}{2} & 1 \end{array}\right] R_2 = -2r_2 \sim \left[\begin{array}{cc|cc} 1 & -\frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & 1 & -3 & -2 \end{array}\right] R_1 = \tfrac{1}{4}r_2 + r_1 \sim \left[\begin{array}{cc|cc} 1 & 0 & -1 & -\frac{1}{2} \\ 0 & 1 & -3 & -2 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & -\frac{1}{2} \\ -3 & -2 \end{bmatrix}$$

$$\underline{X} = A^{-1}B = \begin{bmatrix} -1 & -\frac{1}{2} \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ 14 \end{bmatrix} = \begin{bmatrix} -7 \\ -28 \end{bmatrix}$$

$\boxed{\text{soln: } x = -7,\ y = -28}$

8. Find the function $y = ax^2 + bx + c$ whose graph contains the points $(1, 2)$, $(-2, -7)$, and $(2, -3)$.

We want

$$\begin{cases} 2 = a(1) + b(1) + c \\ -7 = 4a + (-2)b + c \\ -3 = 4a + 2b + c \end{cases} \Rightarrow \begin{array}{c} \begin{array}{cccc} a & b & c & y \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 4 & -2 & 1 & -7 \\ 4 & 2 & 1 & -3 \end{array}\right] \end{array} \begin{array}{l} \\ R_2 = -4r_1 + r_2 \\ R_3 = -4r_1 + r_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -6 & -3 & -15 \\ 0 & -2 & -3 & -11 \end{array}\right] R_2 = -\tfrac{1}{6}r_2 \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & \frac{1}{2} & \frac{5}{2} \\ 0 & -2 & -3 & -11 \end{array}\right] \begin{array}{l} R_1 = -r_2 + r_1 \\ \\ R_3 = +2r_2 + r_3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & -2 & -6 \end{array}\right] R_3 = -\tfrac{1}{2}r_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} R_1 = -\frac{1}{2}r_3 + r_1 \\ R_2 = -\frac{1}{2}r_3 + r_2 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$\therefore a = -2$

$b = 1$

$c = 3$

$\therefore$ the equation is

$\boxed{y = -2x^2 + x + 3}$