Math 1120F - Fall 2014 Name(s): KEY
Worksheet 8 - §1.2, 1.3, 1.4

Name(s):

Work with partners in groups of 2-4. **This is required.**

1. Solve the following equations in $\mathbb{R}$. *each of these can be solved multiple ways

(a) $x^2 - \frac{1}{2}x - \frac{3}{16} = 0$

$$x = \frac{+\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4(-\frac{3}{16})}}{2}$$

$$= \frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{3}{4}}}{2}$$

$$= \frac{\frac{1}{2} \pm 1}{2}$$

$$x = \frac{1}{4} \pm \frac{1}{2}$$

$$\boxed{x = -\frac{1}{4}, \frac{3}{4}}$$

(b) $x^2 + 4x = 21$

$$x^2 + 4x - 21 = 0$$

$$(x-3)(x+7) = 0$$

$x - 3 = 0$ or $x + 7 = 0$

$x = 3$ $\quad$ $x = -7$

$$\boxed{x = 3, -7}$$

(c) $\dfrac{x}{x-2} + \dfrac{2}{x+1} = \dfrac{7x+1}{x^2-x-2}$ $\quad$ $D = \{x \mid x \neq -1, x \neq 2\}$

$$\frac{x}{x-2} + \frac{2}{x+1} = \frac{7x+1}{(x+1)(x-2)}$$

$$x(x+1) + 2(x-2) = 7x + 1$$

$$x^2 + x + 2x - 4 = 7x + 1$$

$$x^2 - 4x - 5 = 0$$

$$(x+1)(x-5) = 0$$

$x + 1 = 0$ or $x - 5 = 0$

$x = -1$ $\quad$ $x = 5$

not in domain

$$\boxed{x = 5}$$

2. A ball is thrown vertically upward from a building which is 96 feet high with an initial velocity of 80 feet per second. The distance s (in feet) of the ball from the ground after t seconds is given by $s = 96 + 80t - 16t^2$. After how many seconds will the ball hit the ground?

The ball will hit the ground when its distance is zero. Solve:

$96 + 80t - 16t^2 = 0$

$16t^2 - 80t - 96 = 0$

$t^2 - 5t - 6 = 0$

$(t+1)(t-6) = 0$

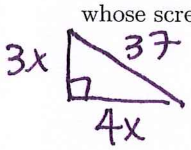
$\Rightarrow t+1=0$ or $t-6=0$

$t=-1$ $\quad$ $t=6$

Discard negative time

$\boxed{\therefore \text{ the ball will hit the ground after 6 seconds}}$

3. The screen size of a television is determined by the length of the diagonal of the rectangular screen. Traditional TVs come in a 4:3 format, meaning the ratio of the length to width of the screen is 4 to 3. What is the area of a 37-inch traditional TV screen? What is the area of the 37-inch LCD TV whose screen is in a 16:9 format?



$37 = \sqrt{(4x)^2 + (3x)^2} = \sqrt{16x^2 + 9x^2} = \sqrt{25x^2} = 5x$

$x = \frac{37}{5}$

$\therefore$ length $= 4\left(\frac{37}{5}\right) = 29.6$

width $= 3\left(\frac{37}{5}\right) = 22.2$

$\therefore \text{area}_{4:3} = (29.6)(22.2) = \boxed{657.12 \text{ in}^2}$

* The area of the 16:9 TV can be solved similarly for a total area of 584.97 in^2

4. Without solving, describe the type of solutions expected for the following quadratic functions:

(a) $3x^2 - 3x + 4 = 4$ $\quad b^2 - 4ac = 9 - 4(3)(4) < 0 \therefore$ two complex conjugates

(b) $2x^2 - 4x + 1 = 0$ $\quad b^2 - 4ac = 16 - 4(2)(1) > 0 \therefore$ two distinct real

(c) $x^2 + 6 = 2x$ $\quad b^2 - 4ac = 2 - 4(6) < 0 \therefore$ two complex conjugates

(d) $9x^2 - 12x + 4 = 0$ $\quad b^2 - 4ac = 144 - 4(9)(4) = 0 \therefore$ one real

5. Write in standard $a + bi$ form.

(a) $(-8 + 2i) - (2 - i)$

$= -8 + 2i - 2 + i = \boxed{-10 + 3i}$

(b) $\dfrac{10}{3 - 4i}$ $= \dfrac{10}{3-4i} \cdot \dfrac{3+4i}{3+4i} = \dfrac{30+40i}{9+16} = \dfrac{30+40i}{25} = \dfrac{30}{25} + \dfrac{40}{25} i$

$= \boxed{\dfrac{6}{5} + \dfrac{8}{5} i}$

(c) $\dfrac{2 + i}{i}$ $= \dfrac{2+i}{i} \cdot \dfrac{-i}{-i}$

$= \dfrac{-2i - i^2}{1} = \boxed{1 - 2i}$

(d) $(1 + i)^2$

$(1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = \boxed{2i}$

(e) $i^2 3$

$i^2 3 = (-1)3 = \boxed{-3}$

6. Find **all** solutions to the following equations.

(a) $x^2 - 6x + 13 = 0$

$x = \dfrac{+6 \pm \sqrt{36 - 4(13)}}{2}$

$= \dfrac{6 \pm \sqrt{-16}}{2}$

$x = \dfrac{6 \pm 4i}{2}$

$\boxed{x = 3 \pm 2i}$

(b) $x^3 = 27$

$x^3 - 27 = 0$

$(x - 3)(x^2 + 3x + 9) = 0$

$x - 3 = 0$ $\quad x^2 + 3x + 9 = 0$

$x = 3$

$x = \dfrac{-3 \pm \sqrt{9 - 4(9)}}{2} = \dfrac{-3 \pm \sqrt{-27}}{2}$

$= \dfrac{-3 \pm i 3\sqrt{3}}{2}$

$\boxed{x = 3, -\frac{3}{2} \pm \frac{3}{2} i \sqrt{3}}$

(c) $x^4 + 3x^2 - 4 = 0$

Let $u = x^2$.

$u^2 + 3u - 4 = 0$

$(u - 1)(u + 4) = 0$

$u = 1$ $\quad u = -4$

$x^2 = 1$ $\quad x^2 = -4$

$x = \pm 1$ $\quad x = \pm 2i$

$\boxed{x = \pm 1, \pm 2i}$

7. Solve the following expressions.

(a) $3+\sqrt{3x+1}=x$

$\sqrt{3x+1}=x-3$

$3x+1=x^2-6x+9$

$0=x^2-9x-8$

$0=(x+1)(x-8)$

$x=-1, 8$

Domain: $3x+1\geqslant 0$

$3x\geqslant -1$

$x\geqslant -\frac{1}{3}$

$\boxed{\therefore \text{ sol}^{\underline{n}} \text{ is } x=8}$

(b) $\sqrt{3x+7}+\sqrt{x+2}=1$

Domain:

$3x+7\geqslant 0$

$x\geqslant -\frac{7}{3}$

$x+2\geqslant 0$

$x\geqslant -2$

$\sqrt{3x+7}=1-\sqrt{x+2}$

$3x+7=1-2\sqrt{x+2}+(x+2)$

$2\sqrt{x+2}=1+x+2-3x-7$

$2\sqrt{x+2}=-2x-4$

$\sqrt{x+2}=-x-2$

$x+2=x^2+4x+4$

$0=x^2+3x+2$

$0=(x+1)(x+2)$

$x=-1,-2$

$\boxed{\therefore \text{ sol}^{\underline{n}} \text{ is } x=-2}$

(c) $\sqrt{3-2\sqrt{x}}=\sqrt{x}$

$3-2\sqrt{x}=x$

$-2\sqrt{x}=x-3$

$4x=x^2-6x+9$

$0=x^2-10x+9$

$0=(x-1)(x-9)$

$x=1, x=9$

$\therefore \boxed{\text{sol}^{\underline{n}} \text{ is } x=1}$

Domain: $3-2\sqrt{x}\geqslant 0$

$-2\sqrt{x}\geqslant -3$

$\sqrt{x}\leq \frac{3}{2}$

$x\leq \frac{9}{4}$

and $x\geqslant 0$ so

$0\leq x\leq \frac{9}{4}$

(d) $(x+2)^2+7(x+2)=12$

Let $w=x+2$.

$u^2+7u-12=0$

$u=\frac{-7\pm\sqrt{49-4(-12)}}{2}$

$u=\frac{-7\pm\sqrt{97}}{2}$

$x+2=\frac{-7\pm\sqrt{97}}{2}$

$\boxed{x=\frac{-11\pm\sqrt{97}}{2}}$