Math 1140F - Exam 1

Name: KEY

Monday, September 8, 2014
Time: 50 minutes
Instructor: Brittany Cuchta

Instructions:

- Do not open the exam until I say you may.
- All cell phones and other electronic noisemaking devices must be turned off or completely silenced (i.e., not on vibrate) for the duration of the exam.
- **No calculators** are allowed on the exam.
- Failure to follow directions specific to a problem will result in the loss of points.
- Circle or box your final answer where appropriate.
- Show **all** work. Full credit will only be given if work is shown which **fully and clearly** justifies your answer.
- Answers must be exact (like $\sqrt{2}$), not approximate (like 1.414), unless a problem specifically indicates otherwise.
- All final answers must be simplified unless otherwise specified. **Rationalization is not required unless otherwise specified.**
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

Page:	1	2	3	4	5	Total
Points:	26	22	16	22	14	100
Score:						

1. (7 points) Is $x + \frac{1}{3}$ a factor of $3x^4 + x^3 - 3x + 1$?

$$\begin{array}{r|rrrrr} -\frac{1}{3} & 3 & 1 & 0 & -3 & 1 \\ & \downarrow & -1 & 0 & 0 & 1 \\ \hline & 3 & 0 & 0 & -3 & 2 \end{array}$$

Circle One: Yes (No)

2. (10 points) Find all real solutions:

$$\frac{x}{x^2-9} = \frac{4}{x+3} + \frac{3}{x^2-9}$$

$D = \{x \mid x \neq \pm 3\}$

$$\frac{x}{x^2-9} = \frac{4}{x+3} + \frac{3}{x^2-9}$$

$\therefore$ no solution

$$x = 4(x-3) + 3$$

$$x = 4x - 12 + 3$$

$$9 = 3x$$

$$3 = x \quad \text{but 3 is not in the domain!}$$

3. (9 points) Find all real solutions:

$$2 + \sqrt{12 - 2x} = x$$

$$\sqrt{12-2x} = x - 2$$

$$12 - 2x = x^2 - 4x + 4$$

$$0 = x^2 - 2x - 8$$

$$0 = (x+2)(x-4)$$

So, either

$x + 2 = 0$ or $x - 4 = 0$

$x = -2$ $\quad$ $x = 4$

Check:

$x = -2$: $2 + \sqrt{12 - (2)(-2)} = 2 + \sqrt{16} = 6 \neq -2$

$x = 4$ $2 + \sqrt{12 - 2(4)} = 2 + \sqrt{4} = 4 = 4$ ✓

Soln: $\{4\}$

4. (10 points) Find **all** solutions to $x^3 - 8 = 0$.

$(x-2)(x^2+2x+4)=0$

$x-2=0$ or $x^2+2x+4=0$

$x=2$

$x=\frac{-2\pm\sqrt{4-4(4)}}{2}=\frac{-2\pm\sqrt{-12}}{2}=\frac{-2\pm 2i\sqrt{3}}{2}$

$\{2,-1\pm i\sqrt{3}\}$

5. Let $P_1=(-1,3)$ and $P_2=(4,-7)$.

(a) (3 points) Find the midpoint of P_1 and P_2.

$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-1+4}{2},\frac{3-7}{2}\right)$

$=\left(\frac{3}{2},-2\right)$

(b) (3 points) Find the distance between P_1 and P_2.

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(4+1)^2+(-7-3)}$

$=\sqrt{25+100}=\sqrt{125}=5\sqrt{5}$

(c) (4 points) Find the equation of the line containing P_1 and P_2.

$m=\frac{y_2-y_1}{x_2-x_1}=\frac{3+7}{-1-4}=\frac{10}{-5}=-2$

$y-y_1=m(x-x_1)$

$y-3=-2(x+1)$

$y=-2x-2+3$

$y=-2x+1$

6. (2 points) What is the slope of the line perpendicular to the line $2x-y=3$?

$2x-y=3$

$y=2x-3$

$\Rightarrow m_1=2$

$m_1m_2=-1$

$2m_2=-1$

$m_2=-\frac{1}{2}$

7. Solve the following in the real numbers. Give your answers in interval notation.

(a) (6 points) $3 - |x+1| < \frac{1}{2}$

$-|x+1| < -\frac{5}{2}$

$|x+1| > \frac{5}{2}$

$x+1 > \frac{5}{2}$ or $x+1 < -\frac{5}{2}$

$x > \frac{3}{2}$ or $x < -\frac{7}{2}$

$\boxed{(-\infty, -\frac{7}{2}) \cup (\frac{3}{2}, \infty)}$

(b) (6 points) $\left|\frac{2x-3}{2} + \frac{1}{3}\right| \leq 1$

$-1 \leq \frac{2x-3}{2} + \frac{1}{3} \leq 1$

$-\frac{4}{3} \leq \frac{2x-3}{2} \leq \frac{2}{3}$

$-\frac{8}{3} \leq 2x-3 \leq \frac{4}{3}$

$\frac{1}{3} \leq 2x \leq \frac{13}{3}$

$\frac{1}{6} \leq x \leq \frac{13}{6}$

$\boxed{[\frac{1}{6}, \frac{13}{6}]}$

8. (4 points) Write $\frac{2+3i}{1-i}$ in $a+bi$ form.

$\frac{2+3i}{1-i} = \frac{2+3i}{1-i} \cdot \frac{1+i}{1+i} = \frac{2+2i+3i+3i^2}{2}$

$= \frac{2+5i-3}{2}$

$= \frac{-1+5i}{2} = \boxed{-\frac{1}{2} + \frac{5}{2}i}$

9. Consider the following circle equation:

$$x^2 + y^2 - 2x - 4y - 4 = 0$$

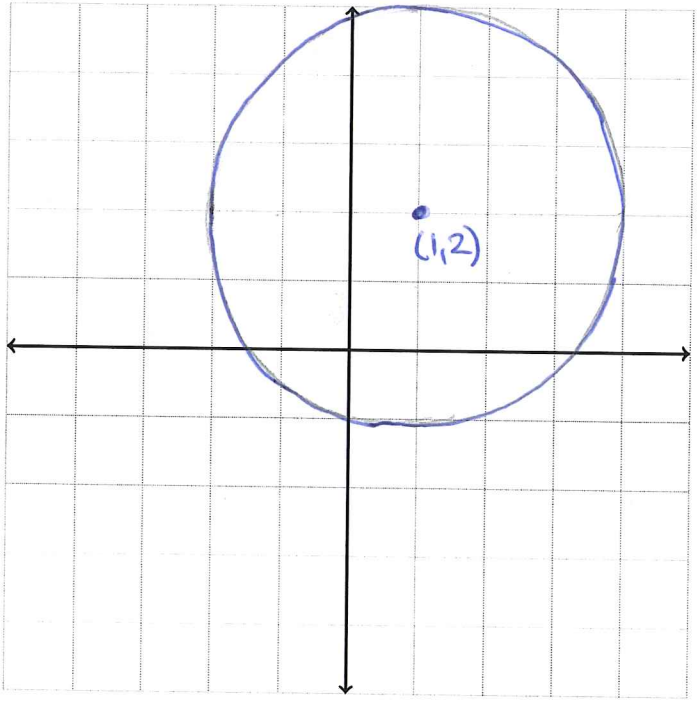
(a) (12 points) Write the equation in standard form.

$x^2 - 2x + y^2 - 4y = 4$

$(x^2 - 2x + 1) + (y^2 - 4y + 4) = 4 + 1 + 4$

$(x-1)^2 + (y-2)^2 = 9$

(b) (3 points) Graph the circle. Clearly label the center.



10. (7 points) Test $y = \dfrac{3x}{x^2+9}$ for any symmetry. List any intercepts.

x-symm: $-y = \dfrac{3x}{x^2+9}$ ✗

y-symm: $y = \dfrac{3(-x)}{(-x)^2+9}$ ✗

origin symm: $-y = \dfrac{3(-x)}{(-x)^2+9}$

$-y = \dfrac{-3x}{x^2+9}$

$y = \dfrac{3x}{x^2+9}$ ✓

y-int: $y = \dfrac{3(0)}{0^2+9} = 0$

x-int: $0 = \dfrac{3x}{x^2+9}$

$0 = 3x$

$x = 0$

Intercepts: (0,0)

x-axis symmetry: Yes (No)

y-axis symmetry: Yes (No)

origin symmetry: (Yes) No

11. (7 points) At 10am, Danny is asked by his father to weed the garden. Danny takes 5 hours to do it by himself. His brother Mike takes 6 hours by himself. If Mike and Danny have a golfing reservation at 1pm, assuming not gain or loss in efficiency, will they be able to finish in time?

$$\frac{1}{5} + \frac{1}{6} = \frac{1}{t}$$

$$\frac{6}{30} + \frac{5}{30} = \frac{1}{t}$$

$$\frac{11}{30} = \frac{1}{t}$$

$$t = \frac{30}{11} < 3$$

Since there are three hours between 10am & 1pm and it will take them less than three hours to finish

yes, they will be done in time

12. (7 points) How much water must be evaporated from 240 gallons of a 3% salt solution to make a 5% salt solution?

$$240(0.03) = (240 - x)(0.05)$$

$$(240)3 = (240 - x)5$$

$$720 = 1200 - 5x$$

$$5x = 480$$

$$x = 96$$

∴ 96 gallons will need to be evaporated.