Math 1140F - Exam 2

Name: KEY

Wednesday, September 17, 2014
Time: 50 minutes
Instructor: Brittany Cuchta

Instructions:

- Do not open the exam until I say you may.
- All cell phones and other electronic noisemaking devices must be turned off or completely silenced (i.e., not on vibrate) for the duration of the exam.
- **No calculators** are allowed on the exam.
- Failure to follow directions specific to a problem will result in the loss of points.
- Circle or box your final answer where appropriate. Put your final answer in the provided space when available.
- Show **all** work. Full credit will only be given if work is shown which **fully and clearly** justifies your answer.
- Answers must be exact (like $\sqrt{2}$), not approximate (like 1.414), unless a problem specifically indicates otherwise.
- All final answers must be simplified unless otherwise specified. **Rationalization is not required unless otherwise specified.**
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

Page:	1	2	3	4	5	6	Total
Points:	19	15	20	10	26	10	100
Score:							

1. Determine whether the relation is a function or not. Circle your answer.

(a) (2 points) $\{(-1,2);(0,5);(3,3);(0,6)\}$ Circle One: Yes No

(b) (2 points) $\{(1,4);(2,3);(5,3);(4,0)\}$ Circle One: Yes No

2. Consider the function $f(x) = \frac{2x}{x-3}$.

(a) (3 points) Is the point $(-1, \frac{1}{2})$ on the graph of $f(x)$?

$f(-1) = \frac{2(-1)}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$

Circle One: Yes No

(b) (3 points) If $f(x) = 1$, what is x?

$1 = \frac{2x}{x-3}$

$x-3 = 2x$

$x = -3$

Answer: $x=-3$

3. Find the domain of each function. State your answer in set notation.

(a) (3 points) $f(x) = \frac{2x}{x^2-9}$

Domain: $\{x \mid x \neq \pm 3\}$

(b) (3 points) $g(x) = \sqrt{-2x-4}$

$-2x-4 \geq 0$

$-4 \geq 2x$

$-2 \geq x$

Domain: $\{x \mid x \leq -2\}$

(c) (3 points) $h(x) = \frac{\sqrt{x+3}}{x-1}$

$x+3 \geq 0$

$x \geq -3$

$x-1=0$

$x=1$

Domain: $\{x \mid x \geq -1$ and $x \neq 1\}$

4. Given the piecewise-defined function

$$G(x) = \begin{cases} x^2 & \text{if } -2 \le x \le 2 \\ 2x - 1 & \text{if } x > 2 \end{cases}$$

(a) (2 points) State the domain.

Domain: $[-2, \infty)$

(b) (3 points) Locate any intercepts. If there are none, state so.

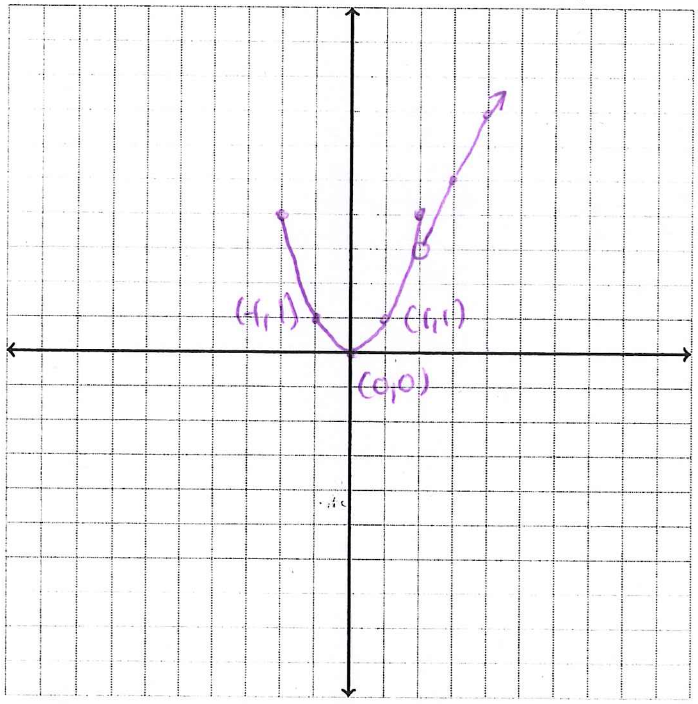
x-int: $0 = x^2 \Rightarrow (0,0)$

$2x - 1 = 0$

$x = \frac{1}{2}$, not in $x > 2$

Intercepts: $(0,0)$

(c) (6 points) Graph the function. Be sure to label three points.



(d) (2 points) Based on the graph, state the range.

Range: $[0, \infty)$

(e) (2 points) Is $G(x)$ continuous on its domain? Circle your answer and state why or why not.

there is a jump at $x = 2$.

Circle One: Yes No

5. (6 points) Determine ***algebraically*** whether the function is either, odd, or neither. Be sure to show work.

$$f(x) = \frac{x^3 + x}{-3x^4 - 7}$$

$f(-x) = \dfrac{(-x)^3 + (-x)}{-3(-x)^4 - 7} = \dfrac{-x^3 - x}{-3x^4 - 7} = -\dfrac{(x^3+x)}{-3x^4-7} = -f(x)$

Circle One: Even Odd Neither

6. Consider the quadratic function $f(x) = -2x^2 + x + 6$. Answer the following questions.

(a) (3 points) Determine if the quadratic function opens up or down.

Circle One: Opens Up Opens Down

(b) (4 points) Find the vertex.

$X = \dfrac{-b}{2a} = \dfrac{-1}{2(-2)} = \dfrac{1}{4}$

$f(\frac{1}{4}) = -2(\frac{1}{4})^2 + (\frac{1}{4}) + 6$

$= -\dfrac{2}{16} + \dfrac{1}{4} + 6$

$= -\dfrac{1}{8} + \dfrac{2}{8} + \dfrac{48}{8} = \dfrac{49}{8}$

Vertex: $\left(\frac{1}{4}, \frac{49}{8}\right)$

(c) (3 points) Find the axis of symmetry.

Axis: $X = \frac{1}{4}$

(d) (4 points) Find the x-intercepts, if any. If there are none, state so.

$0 = -2x^2 + x + 6$

$0 = 2x^2 - x - 6$

$0 = (2x+1)(x-3)$

$2x+1=0 \qquad x-3=0$

$x = -\frac{1}{2} \qquad x = 3$

x-intercepts: $\left(\frac{1}{2}, 0\right), (3, 0)$

7. (10 points) Graph the function $H(x) = -\sqrt{x+3}+1$ using the techniques of shifting, compressing, stretching, and reflecting. Fill in the following steps with the equation you will graph and graph each step on one grid. Be sure to label at least three points on your final graph.

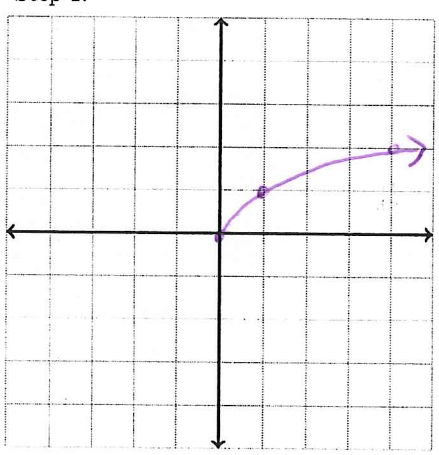
Step 1: begin with $\sqrt{x}$

Step 2: $-\sqrt{x}$

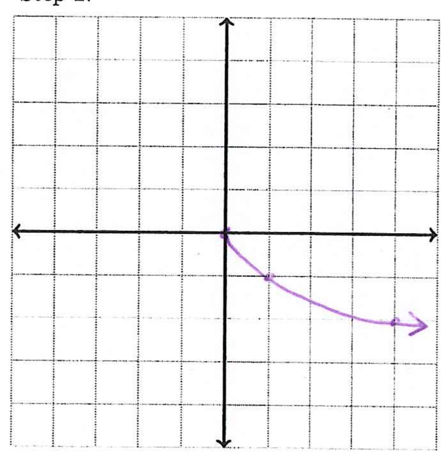
Step 3: $-\sqrt{x+3}$

Step 4: $-\sqrt{x+3}+1$

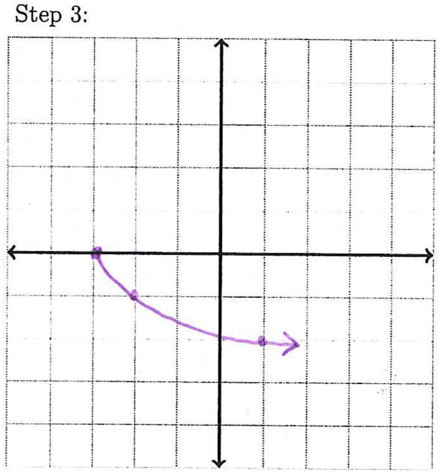
Step 1:



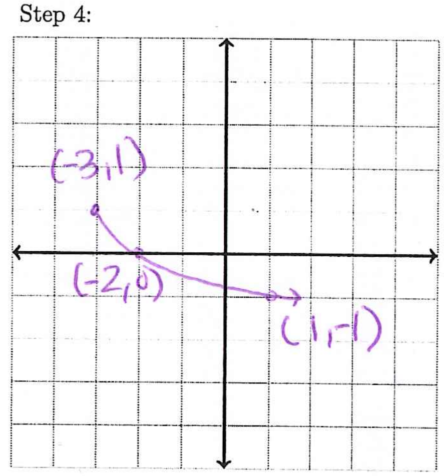
Step 2:



Step 3:



Step 4:



8. Solve each inequality. Express your answers in interval notation. Show all work.

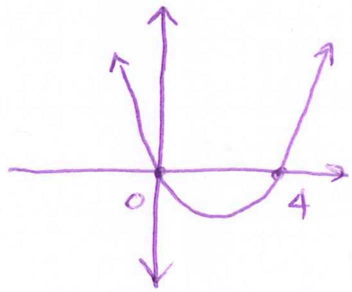
(a) (8 points) $x^2 - 4x > 0$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$, $x = 4$

opens up



Answer: $(-\infty, 0) \cup (4, \infty)$

(b) (8 points) $3x - 10 \leq -x^2$

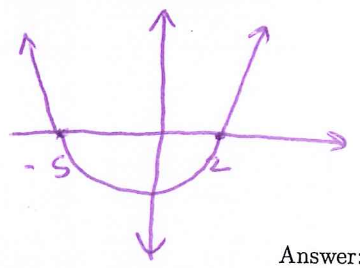
$3x - 10 + x^2 \leq 0$

$x^2 + 3x - 10 = 0$

$(x-2)(x+5) = 0$

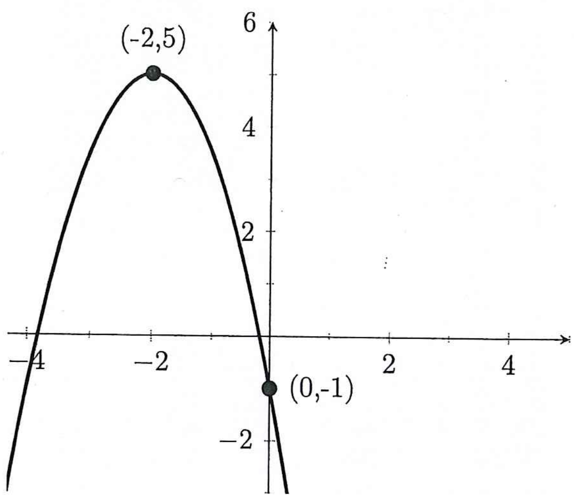
$x = 2$, $x = -5$

opens up



Answer: $[-5, 2]$

9. (10 points) Determine the quadration function whose graph is given. **Show work.**



$(h, k) = (-2, 5)$

$f(x) = a(x-h)^2 + k$

$f(x) = a(x+2)^2 + 5$

$f(0) = a(0+2)^2 + 5 = -1$

$4a + 5 = -1$

$4a = -6$

$a = -\frac{3}{2}$

Answer: $f(x) = -\frac{3}{2}(x+2)^2 + 5$

10. David has 300 feet of fencing available to enclose a rectangular field.

(a) (5 points) Express the area A of the rectangle as a function of x, where x is the length of the rectangle.

$2l+2w=300$

$l+w=150$

$w=150-l$

$A(x)=xw$

$=x(150-x)$

$\boxed{A(x)=150x-x^2}$

l will be represented as x.

(b) (3 points) For what value of x is the area the largest?

$x=\frac{-b}{2a}=-\frac{150}{2(-1)}=\boxed{75 \text{ feet}}$

(c) (2 points) What is the maximum area?

$A(75)=150(75)-(75)^2$

$=11250-5625$

$=\boxed{5625 \text{ feet squared}}$

$$\begin{array}{r} 75 \\ 150 \\ \hline 3750 \\ 75 \\ \hline 11250 \end{array} \qquad \begin{array}{r} 75 \\ 75 \\ \hline 375 \\ 525 \\ \hline 5625 \end{array}$$

$$\begin{array}{r} 11250 \\ 5625 \\ \hline 5625 \end{array}$$