Math 1140F - Exam 3

Name: KEY

Thursday, September 25, 2014
Time: 50 minutes
Instructor: Brittany Cuchta

Instructions:

- Do not open the exam until I say you may.
- All cell phones and other electronic noisemaking devices must be turned off or completely silenced (i.e., not on vibrate) for the duration of the exam.
- **No calculators** are allowed on the exam.
- The exam *must* be taken in pencil. Using a pen on the exam will result in the loss of points.
- Failure to follow directions specific to a problem will result in the loss of points.
- Circle or box your final answer where appropriate. Put your final answer in the provided space when available. Failure to do so will result in points being deducted.
- Show **all** work. Full credit will only be given if work is shown which **fully and clearly** justifies your answer. I reserve the right to not grade a problem which I cannot read.
- Answers must be exact (like $\sqrt{2}$), not approximate (like 1.414), unless a problem specifically indicates otherwise.
- All final answers must be simplified unless otherwise specified. **Rationalization is not required unless otherwise specified.**
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

Page:	1	2	3	4	5	Total
Points:	24	22	26	10	18	100
Score:						

1. (12 points) Construct a polynomial $f(x)$ with the following characteristics:
   (a) Zeros: -2 (multiplicity 3), 2 (multiplicity 1), 0 (multiplicity 2)
   (b) Degree: 6
   (c) Contains the point: (1, 8)

$f(x) = ax^2(x+2)^3(x-2)$

$f(1) = a(2)^3(-1) = 8$

$-27a = 8$

$a = -\frac{8}{27}$

Polynomial $f(x) = \underline{-\frac{8}{27}x^2(x+2)^3(x-2)}$

2. (6 points) The graph of the polynomial $r(x) = x^2(x-2)(x-3)^3(x-4)^6$
   (a) touches the x-axis at $x =$ 0, 4
   (b) crosses the x-axis at $x =$ 2, 3

3. (6 points) Find a polynomial $g(x)$ of degree 4 with zeros 3 (multiplicity 2) and i.

$g(x) = a(x-3)^2(x-i)(x+i)$

$g(x) = a(x-3)^2(x^2+1)$

let $a=1$.

Polynomial $g(x) = \underline{(x-3)^2(x^2+1)}$

4. For the polynomial function $H(x) = x^4 - 2x^3 - 4x^2 - 8x - 32$

(a) (4 points) Is $x = 1$ a zero for $H(x)$? Show work and circle your answer.

$H(1) = 1 - 2 - 4 - 8 - 32 \neq 0$

Circle One: Yes (No)

(b) (14 points) Use the **Rational Root Theorem** to find all **real** zero(s) of $H(x)$. Using any other method will result in no points being given.

$p = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$

$q = \pm 1$

$\Rightarrow \frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & -4 & -8 & -32 \\ & \downarrow & -2 & 8 & -8 & 32 \\ \hline & 1 & -4 & 4 & -16 & 0 \end{array}$$

$\therefore x = -2$ is a zero

Depressed eqn: factoring:

$x^3 - 4x^2 + 4x - 16 = 0$

$x^2(x-4) + 4(x-4) = 0$

$(x^2+4)(x-4) = 0$

$\therefore x = 4$ is a zero

$x^2 + 4 = 0$ no real soln

Real Zero(s): $x = -2, x = 4$

(c) (4 points) Find all **complex** zeros of $H(x)$.

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

Complex Zero(s): $x = +2i, x = -2i$

5. Given the function $T(x) = \dfrac{x^2 - 64}{x}$

(a) (3 points) State the domain. Give your answer in interval notation.

Domain: $(-\infty, 0) \cup (0, \infty)$

(b) (6 points) List the x and y intercepts. If there is none, state so in the provided space.

x-int: $0 = x^2 - 64$

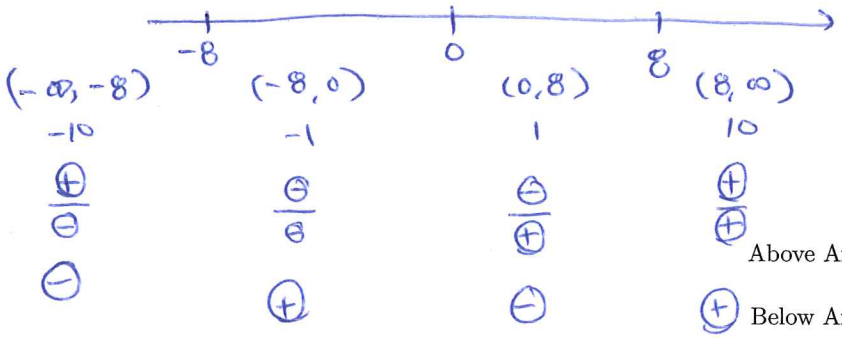
$x^2 = 64$

$x = \pm 8$

x-intercept(s): $(8, 0), (-8, 0)$

y-intercept: DNE

(c) (6 points) State on which intervals the graph is above the x-axis and below the x-axis. Use the table method discussed in class.



Above Axis: $(-8, 0) \cup (8, \infty)$

Below Axis: $(-\infty, -8) \cup (0, 8)$

(d) (11 points) Find all vertical, horizontal or oblique asymptotes. If the result doesn't exist, state so in the space provided.

Vertical: $x = 0$

Since $\deg(p(x)) = 2 = 1 + 1 = 1 + \deg(q(x))$

asymptote is oblique

$$x \overline{)\, x^2 - 64}$$

$$x^2$$

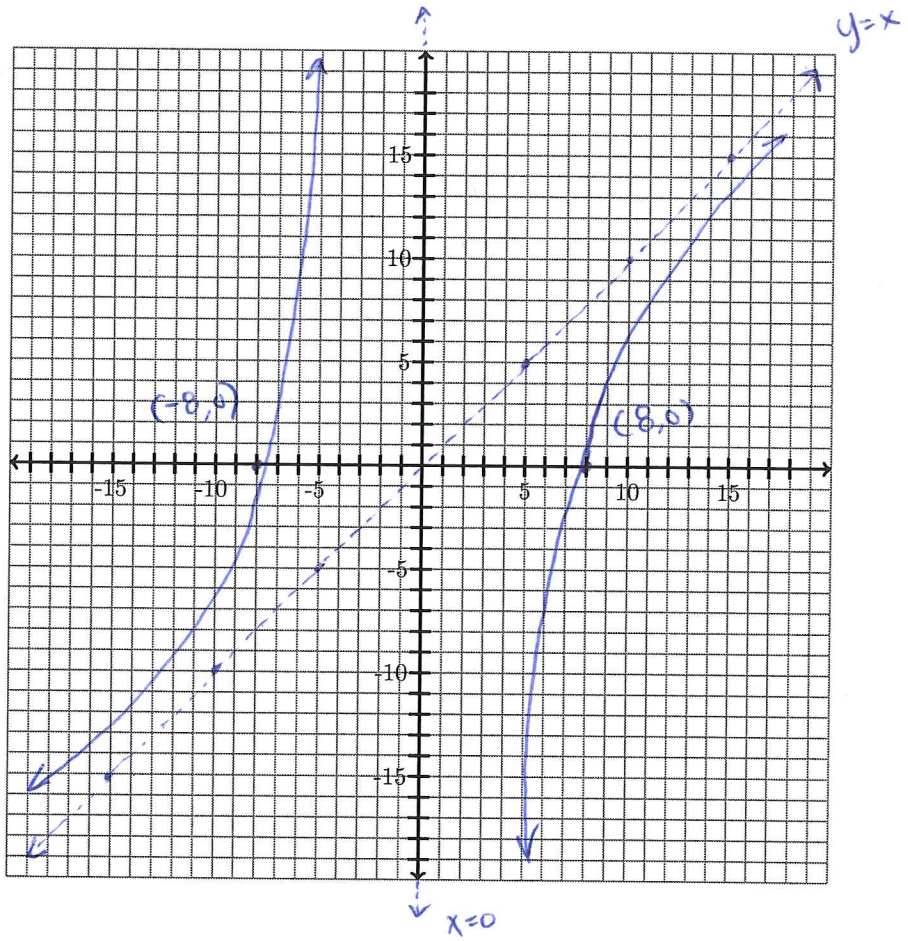
$$-64$$

Vertical Asymptote(s): $x = 0$

Horizontal Asymptote: DNE

Oblique Asymptote: $y = x$

(e) (6 points) Sketch the graph of $T(x)$. Be sure to label any intercepts and asymptotes.



6. (4 points) Solve the inequality $\frac{x^2-64}{x} \geq 0$. Give your answer in interval notation.

from graph above or from 5(c),

Answer: $(-8,0) \cup [8,\infty)$

7. (12 points) Solve the following inequality **algebraically**. Give your answer in interval notation.

$$\frac{4x+5}{x+2} \geq 3$$

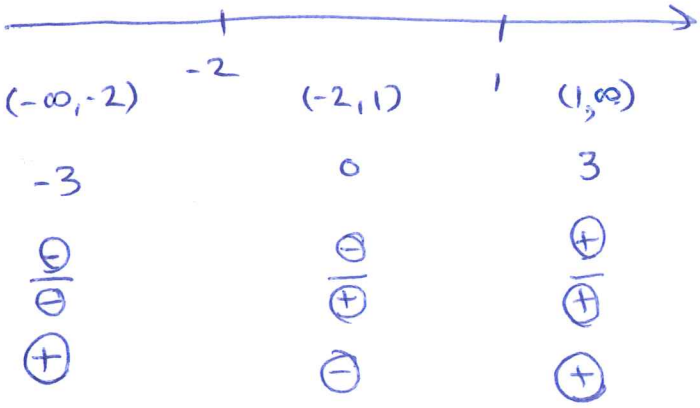
$\frac{4x+5-3x-6}{x+2} \geq 0$

$\frac{x-1}{x+2} \geq 0$

$f(x) = \frac{x-1}{x+2}$

zeros: $x = 1$

asymptotes: $x = -2$



Answer: $(-\infty, -2) \cup [1, \infty)$

8. (6 points) Use the **Intermediate Value Theorem** to determine whether or not the polynomial $Q(x) = 4x^4 + 15x^2 - 4$ has a zero in the interval $[0, 1]$. Circle your answer. Failure to show work will result in no points being awarded.

$Q(0) = -4 < 0$

$Q(1) = 4 + 15 - 4 > 0$

By IVT, there must be a zero in $[0, 1]$.

Circle One: (Yes) No