Math 1140F - Exam 4

Name: KEY

Monday, October 6, 2014
Time: 50 minutes
Instructor: Brittany Cuchta

Instructions:

- Do not open the exam until I say you may.
- All cell phones and other electronic noisemaking devices must be turned off or completely silenced (i.e., not on vibrate) for the duration of the exam.
- **No calculators** are allowed on the exam.
- The exam *must* be taken in pencil. Using a pen on the exam will result in the loss of points.
- Failure to follow directions specific to a problem will result in the loss of points.
- Circle or box your final answer where appropriate. Put your final answer in the provided space when available. Failure to do so will result in points being deducted.
- Show **all** work. Full credit will only be given if work is shown which **fully and clearly** justifies your answer. I reserve the right to not grade a problem which I cannot read.
- Answers must be exact (like $\sqrt{2}$), not approximate (like 1.414), unless a problem specifically indicates otherwise.
- All final answers must be simplified unless otherwise specified. **Rationalization is not required unless otherwise specified.**
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

Page:	1	2	3	4	5	Total
Points:	18	15	24	28	15	100
Score:						

1. (10 points) If $f(x) = \dfrac{x}{x+3}$ and $g(x) = \dfrac{27}{x+1}$, find $(f \circ g)(2)$. Give the domain of $(f \circ g)(x)$ in set notation.

$$g(2) = \frac{27}{2+1} = \frac{27}{3} = 9$$

$$f(9) = \frac{9}{9+3} = \frac{9}{12} = \frac{3}{4}$$

$D_g = \{x \mid x \neq -1\}$

$D_f = \{x \mid x \neq -3\}$

$$\frac{27}{x+1} = -3$$

$27 = -3x - 3$

$30 = -3x$

$x = -10$

$(f \circ g)(2) =$ $\frac{3}{4}$

Domain of $(f \circ g)(x)$: $\{x \mid x \neq -1, -10\}$

2. (4 points) Give f and g such that $(f \circ g)(x) = H(x)$.

$$H(x) = \frac{1}{\sqrt{6x+6}}$$

$f(x) =$ $\frac{1}{\sqrt{x}}$

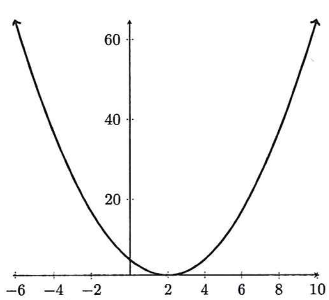
$g(x) =$ $6x+6$

3. Are the following functions one-to-one?

(a) (2 points) $\{(1,3), (2,4), (-2,3), (4,2)\}$

Circle One: Yes No

(b) (2 points)



Circle One: Yes No

4. (11 points) Find the inverse of the following function. Be sure to **check your answer**. Failure to show a check of your solution will result in points being lost. Also state the domain and range **of the inverse function** in set notation.

$$f(x) = \frac{x^2+3}{3x^2}, \quad x > 0$$

$y = \dfrac{x^2+3}{3x^2}, \; x>0$

$3x^2y = x^2+3$

$3x^2y - x^2 = +3$

$x^2(3y-1) = 3$

$x^2 = \dfrac{3}{3y-1}$

$x = \pm\sqrt{\dfrac{3}{3y-1}}$

Domain $f^{-1}(y)$:

$\dfrac{3}{3y-1} \geq 0$

zeros: none

asymptotes: $\frac{1}{3}$

$(-\infty, \frac{1}{3})$ $\quad \frac{1}{3} \quad$ $(\frac{1}{3}, \infty)$

$-1 \qquad\qquad 1$

$\dfrac{+}{-} = - \qquad \dfrac{+}{+} = +$

<u>Check</u>: $f(f^{-1}(y)) = \dfrac{\left(\sqrt{\frac{3}{3y-1}}\right)^2+3}{3\left(\sqrt{\frac{3}{3y-1}}\right)^2} = \dfrac{\frac{3+9y-3}{3y-1}}{\frac{9}{3y-1}} = y \checkmark$

$f^{-1}(f(x)) = \sqrt{\dfrac{3}{3\left(\frac{x^2+3}{3x^2}\right)-1}} = \sqrt{\dfrac{3}{\frac{x^2+3}{x^2} - \frac{x^2}{x^2}}}$

$= \sqrt{x^2} = x \checkmark$

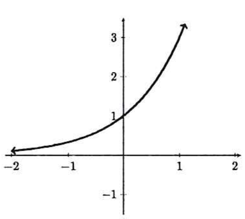
Inverse: $f^{-1}(y) = \sqrt{\dfrac{3}{3y-1}}$

Domain: $\{y \mid y > \frac{1}{3}\}$

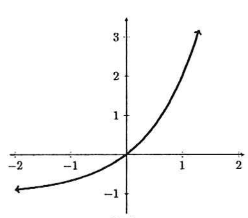
Range: $\{x \mid x > 0\}$

5. (4 points) The equation of an exponential function is given. Select the graph that best represents the function. Clearly circle your answer.

$$f(x) = 3^{x-1}$$



A



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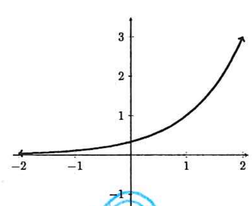

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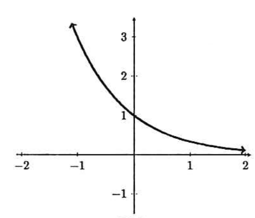

D

6. Solve the following equations. Express **all** powers as factors in logarithms.

(a) (6 points) $3^{x^3} = 9^x$

$3^{x^3} = (3^2)^x$

$3^{x^3} = 3^{2x}$

$x^3 = 2x$

$x^3 - 2x = 0$

$x(x^2-2) = 0$

$x = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

Solution: $\{0, \sqrt{2}, -\sqrt{2}\}$

(b) (8 points) $e^{-2x} = \frac{1}{3}$

$\ln e^{-2x} = \ln \frac{1}{3}$

$-2x = -\ln 3$

$x = \frac{\ln 3}{2}$

Solution: $\{\frac{1}{2}\ln 3\}$

(c) (10 points) $\log_3(x^3+1) = 2$

$\log_3(x^3+1) = 2$

$x^3 + 1 = 3^2$

$x^3 + 1 = 9$

$x^3 - 8 = 0$

$(x-2)(x^2+2x+4) = 0$

$x - 2 = 0$

$x = 2$

$x^2 + 2x + 4 = 0$

no real solutions

Solution: $\{2\}$

7. Solve the following equations.

(a) (12 points) $\log x + \log(x - 21) = 2$

$\log(x(x-21)) = 2$

$x^2 - 21x = 10^2$

$x^2 - 21x = 100$

$x^2 - 21x - 100 = 0$

$\rightarrow (x+4)(x-25) = 0$

$x = -4 \qquad x = 25$

$\log(-4)$ undefined

Solution: $\{25\}$

(b) (10 points) $2^{2x} + 2^x - 12 = 0$

Let $u = 2^x$. Then

$u^2 + u - 12 = 0$

$(u-3)(u+4) = 0$

$u = 3 \qquad u = -4$

$2^x = 3 \qquad 2^x = -4$

$x = \log_2 3 \qquad$ undefined

Solution: $\{\log_2 3\}$

8. (6 points) If $\ln 2 = a$ and $\ln 3 = b$, express $\ln \sqrt[5]{6}$ in terms of a and b

$\ln(\sqrt[5]{6}) = \ln(6^{\frac{1}{5}}) = \frac{1}{5}\ln 6$

$= \frac{1}{5}\ln(2 \cdot 3)$

$= \frac{1}{5}[\ln 2 + \ln 3] = \frac{1}{5}[a+b]$

Solution: $\frac{1}{5}(a+b)$

9. (4 points) Write the following expression as a single logarithm.

$$\log \frac{1}{x} - \log \frac{1}{x^2}$$

$\log \frac{1}{x} - \log \frac{1}{x^2} = \log(x^{-1}) - \log(x^{-2})$

$= \log(x^{-1+2})$

$= \boxed{\log x}$

10. (5 points) Write the following expression as a sum and difference of logarithms. Express powers as factors. All polynomials which appear must be factored *completely.*

$$\log \frac{\sqrt[3]{x^2+1}}{x^2-1}$$

$\log\left[\frac{\sqrt[3]{x^2+1}}{x^2-1}\right] = \log(\sqrt[3]{x^2+1}) - \log(x^2-1)$

$= \frac{1}{3}(\log(x^2+1)) - \log[(x+1)(x-1)]$

$= \boxed{\frac{1}{3}[\log(x^2+1)] - \log(x+1) - \log(x-1)}$

11. (6 points) Given the following function, graph the inverse on the same grid.