Math 1140F – Exam 5

Name: KEY

Thursday, October 16, 2014
Time: 50 minutes
Instructor: Brittany Cuchta

INSTRUCTIONS:

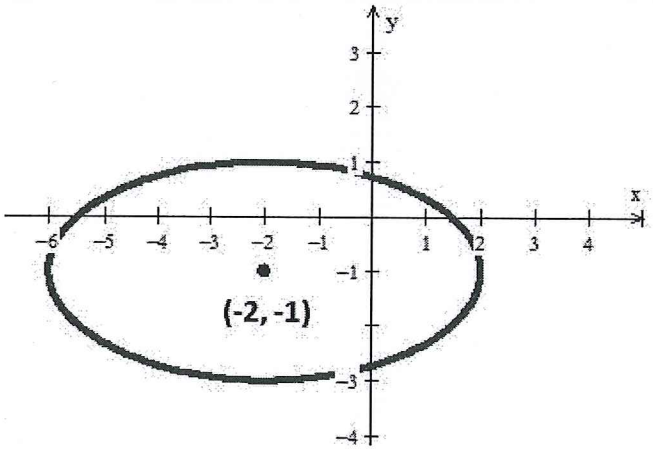
- Do not open the exam until I say you may
- All cell phones and other electronic noisemaking devices must be turned off or completely silenced (i.e., not on vibrate) for the duration of the exam.
- **No calculators** may be used on the exam.
- The exam *must* be taken in pencil. Using a pen on the exam will result in the immediate loss of points.
- Put your final answer in the provided space when available. Failure to do so will result in points being deducted. Circle or box your answer when a space is not provided.
- Failure to follow directions specific to a problem will result in loss of points.
- Show **all** work. Credit will only be given if work is shown which **fully and clearly** justifies your answer. I reserve the right to not grade a problem which I cannot read.
- All final answers must be simplified. **Rationalization is not required**.
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

Page:	**2**	**3**	**4**	**5**	**6**	**Total**
Points:	21	26	17	16	20	100
Score:						

1. (6 pts) Identify each conic. No work is necessary. Your answer is either right or wrong. No partial credit will be awarded. Circle your answer.

a) $x^2+3x-5y-9=0$	ellipse	parabola	hyperbola	none of these
b) $2x^2+5y^2-4x-7y-3=0$	ellipse	parabola	hyperbola	none of these
c) $2x^2-3y^2=12$	ellipse	parabola	hyperbola	none of these

2. (5 pts) Select the function that best represents the given ellipse. Clearly circle your answer.



A. $\frac{(x+2)^2}{16}+\frac{(y+1)^2}{4}=1$

B. $\frac{(x+1)^2}{16}+\frac{(y+2)^2}{4}=1$

C. $\frac{(x-2)^2}{4}+\frac{(y-1)^2}{16}=1$

D. $\frac{(x-2)^2}{16}+\frac{(y-1)^2}{4}=1$

3. (10 pts) Find the determinant of the following matrix.

$$A=\begin{bmatrix} 3 & -2 & -1 \\ 2 & 0 & 5 \\ -4 & 0 & 8 \end{bmatrix}$$

$$\det A=\begin{vmatrix} 3 & -2 & -1 \\ 2 & 0 & 5 \\ -4 & 0 & 8 \end{vmatrix}=3\begin{vmatrix} 0 & 5 \\ 0 & 8 \end{vmatrix}-(-2)\begin{vmatrix} 2 & 5 \\ -4 & 8 \end{vmatrix}+(-1)\begin{vmatrix} 2 & 0 \\ -4 & 0 \end{vmatrix}$$

$=3(0)+2(16+20)-1(0)$

$=2(36)=72$

D = 72

3. (12 pts) Find the value of x using **Cramer's Rule** in the following system:

$$\begin{cases} \frac{1}{2}x + y = -2 \\ x - 2y = 8 \end{cases}$$

$$D = \begin{vmatrix} \frac{1}{2} & 1 \\ 1 & -2 \end{vmatrix} = -1 - 1 = -2$$

$$Dx = \begin{vmatrix} -2 & 1 \\ 8 & -2 \end{vmatrix} = 4 - 8 = -4$$

$$x = \frac{Dx}{D} = \frac{-4}{-2} = 2$$

$x =$ 2

4. (14 pts) Find the center, vertices, and foci of the following equation:

$$9x^2 + 4y^2 - 18x + 16y - 11 = 0$$

$$(9x^2 - 18x) + (4y^2 + 16y) = 11$$

$$9(x^2 - 2x) + 4(y^2 + 4y) = 11$$

$$9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = 11 + 9 + 16$$

$$9(x-1)^2 + 4(y+2)^2 = 36$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$

Center: $(1, -2)$

$a = 3$

$b = 2$

$9 > 4$ so vertical ellipse

Vertices: $(1, -2 \pm 3) = (1, -5), (1, 1)$

$b^2 = a^2 - c^2$

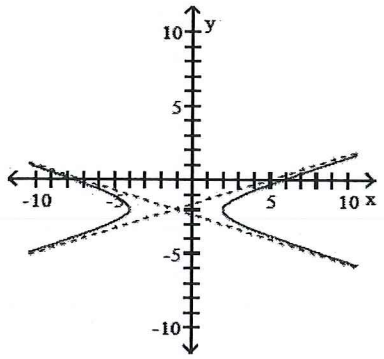
$\Rightarrow c = \sqrt{9-4} = \sqrt{5}$

Foci: $(1, -2 \pm \sqrt{5})$

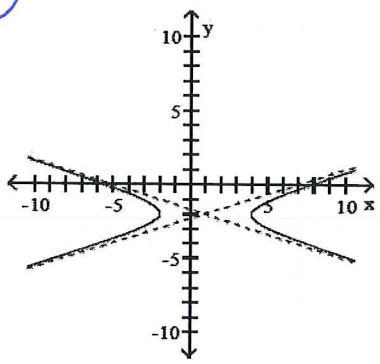
6. (5pts) Select the graph that best represents the following equation. Clearly circle your answer.

$$(x-1)^2 - 9(y+2)^2 = 9$$

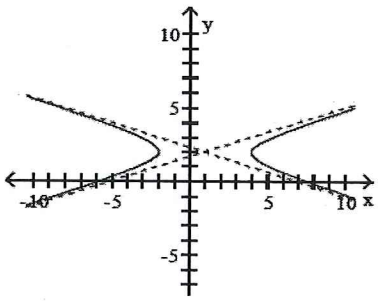
A)



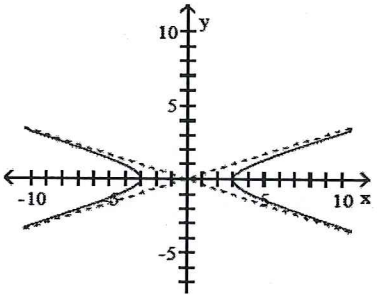
B)



C)



D)



7. (12 pts) Find an equation for the hyperbola with center at $(-4,3)$, one vertex at $(-4,2)$, and one focus at $(-4,0)$.

a. (8pts) Find the equation.

$(h,k) = (-4,3)$

$a = 1$,

$c = 3$

$b^2 = 9 - 1 = 8$

vertical hyperbola so negative is in front of x

Equation: $\frac{(y-3)^2}{1} - \frac{(x+4)^2}{8} = 1$

b. (4pts) Find the Asymptotes.

Asymptotes: $y - 3 = \pm \frac{1}{2\sqrt{2}}(x+4)$

8. (16 pts) Use the following matrices to answer (a) and (b). If the problem cannot be evaluated, explain why.

$$A=\begin{bmatrix}0 & 3 & -2\\ 1 & 2 & 6\end{bmatrix} \quad B=\begin{bmatrix}4 & 1 & 0\\ -2 & 3 & -2\end{bmatrix} \quad C=\begin{bmatrix}4 & 1\\ 6 & 2\\ -2 & 3\end{bmatrix}$$

a) (6 pts) 2A-B

$$2A-B=2\begin{bmatrix}0 & 3 & -2\\ 1 & 2 & 6\end{bmatrix}-\begin{bmatrix}4 & 1 & 0\\ -2 & 3 & -2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 6 & -4\\ 2 & 4 & 12\end{bmatrix}-\begin{bmatrix}4 & 1 & 0\\ -2 & 3 & -2\end{bmatrix}$$

$$=\begin{bmatrix}-4 & 5 & -4\\ 4 & 1 & 14\end{bmatrix}$$

b) (10pts) BC

$$\begin{bmatrix}4 & 1 & 0\\ -2 & 3 & -2\end{bmatrix}\begin{bmatrix}4 & 1\\ 6 & 2\\ -2 & 3\end{bmatrix}=\begin{bmatrix}4(4)+1(6)+0(-2) & 4(1)+1(2)+0(3)\\ -2(4)+3(6)+(-2)(-2) & -2(1)+3(2)+(-2)(3)\end{bmatrix}$$

$2\times 3 \cdot 3\times 2$

expect 2×2

$$=\begin{bmatrix}22 & 6\\ 14 & -2\end{bmatrix}$$

9. (10pts) **Using row reduction techniques,** find the inverse of matrix A. Failure to use row reduction will result in **no points** being awarded.

$$A=\begin{bmatrix}4 & 1\\ 6 & -2\end{bmatrix}$$

$$\left[\begin{array}{cc|cc}4 & 1 & 1 & 0\\ 6 & -2 & 0 & 1\end{array}\right] R_1=\tfrac{1}{4}r_1 \sim \left[\begin{array}{cc|cc}1 & \frac{1}{4} & \frac{1}{4} & 0\\ 6 & -2 & 0 & 1\end{array}\right] R_2=-6r_1+r_2$$

$$\sim \left[\begin{array}{cc|cc}1 & \frac{1}{4} & \frac{1}{4} & 0\\ 0 & -\frac{7}{2} & -\frac{6}{4} & 1\end{array}\right] R_2=-\tfrac{2}{7}r_2 \sim \left[\begin{array}{cc|cc}1 & \frac{1}{4} & \frac{1}{4} & 0\\ 0 & 1 & \frac{3}{7} & -\frac{2}{7}\end{array}\right]$$

$$R_1=-\tfrac{1}{4}r_2+r_1 \left[\begin{array}{cc|cc}1 & 0 & \frac{1}{7} & \frac{1}{14}\\ 0 & 1 & \frac{3}{7} & -\frac{2}{7}\end{array}\right]$$

$$A^{-1}=\begin{bmatrix}\frac{1}{7} & \frac{1}{14}\\ \frac{3}{7} & -\frac{2}{7}\end{bmatrix}$$

10. (10 pts) A 1000-acre farm in Missouri is used to grow corn and soybeans. The cost per acre for raising corn is \$65 and \$45 for soybeans. If \$54320 has been budgeted for costs and all the acreage is to be used, how many acres should be allocated for each crop?

Let x = corn
y = soybeans

$$\begin{cases} x+y=1000 & (1)\\ 65x+45y=54320 & (2)\end{cases}$$

From (1): $y=1000-x$.

Sub into (2):

$$65x+45(1000-x)=54320$$
$$65x+45000-45x=54320$$
$$20x=9320$$
$$x=466$$

with $x=466$,
Sub into (1):
$y=1000-466=534$

∴ 466 acres for corn and 534 acres for soybeans