

You may work in groups of 2-4. Leave answers in exact form and in set notation where appropriate.

1. (2 points) Evaluate  $\frac{4x^3 - 3x^2 - 8x + 4}{x-2}$ . Be sure to write your final answer in a polynomial form.

$$\begin{array}{r} 2 \overline{) 4 \ -3 \ -8 \ 4} \\ \underline{4 \ 5 \ 2 \ 8} \\ \phantom{4} \phantom{5} \phantom{2} \phantom{8} \end{array}$$

$$\frac{4x^3 - 3x^2 - 8x + 4}{x-2} = 4x^2 + 5x + 2 + \frac{8}{x-2}$$

2. (2 points) Fully and clearly derive the quadratic equation.

$$\begin{aligned} ax^2 + bx + c &= 0 \\ ax^2 + bx &= -c \\ x^2 + \frac{b}{a}x &= -\frac{c}{a} \\ x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 &= -\frac{c}{a} + \frac{b^2}{4a^2} \\ \left(x + \frac{b}{2a}\right)^2 &= \frac{-4ac + b^2}{4a^2} \end{aligned}$$

$$\begin{aligned} \Rightarrow \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ x + \frac{b}{2a} &= \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\ x + \frac{b}{2a} &= \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\ \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \end{aligned}$$

3. (2 points) Solve  $\sqrt{10 + 3\sqrt{x}} = \sqrt{x}$  in  $\mathbb{R}$ .

$\sqrt{10 + 3\sqrt{x}} = \sqrt{x}$  square both sides

$$10 + 3\sqrt{x} = x$$

$3\sqrt{x} = x - 10$  square both sides

$$9x = x^2 - 20x + 100$$

$$0 = x^2 - 29x + 100$$

4. (2 points) Solve  $x + \sqrt{x} = 20$  in  $\mathbb{R}$ .

$$x + \sqrt{x} = 20$$

$$x + \sqrt{x} - 20 = 0$$

let  $u = \sqrt{x}$ . Then

$$\Rightarrow u^2 + u - 20 = 0$$

$$(u+5)(u-4) = 0$$

$$u = -5 \text{ or } u = 4$$

$$\sqrt{x} = -5 \quad \sqrt{x} = 4$$

not possible!  $x = 16$

5. (2 points) Solve  $2x^2 - 3x = 1$  in  $\mathbb{R}$  by completing the square.

$$2x^2 - 3x = 1$$

$$x^2 - \frac{3}{2}x = \frac{1}{2}$$

$$x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \frac{1}{2} + \frac{9}{16}$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$\begin{aligned} \Rightarrow x - \frac{3}{4} &= \pm \frac{\sqrt{17}}{4} \\ x &= \frac{3 \pm \sqrt{17}}{4} \end{aligned}$$

$$\boxed{\text{Soln: } \left\{ \frac{3 \pm \sqrt{17}}{4} \right\}}$$

Sol 1:  
{2,5}

$$\begin{aligned} \Rightarrow 0 &= (x-4)(x-25) \\ \Rightarrow x &= 4, 25 \end{aligned}$$

check:  $\sqrt{10 + 3\sqrt{4}} = \sqrt{10 + 3(2)} = \sqrt{16} = 4 \neq 2$   
 $\sqrt{10 + 3\sqrt{25}} = \sqrt{10 + 3(5)} = \sqrt{25} = 5 \checkmark$

Soln: {16}

check:  $16 + \sqrt{16} = 16 + 4 = 20 \checkmark$