

1. (3 points) Analyze the graph of $f(x) = (x-1)^2(x+3)(x+1)$ using the steps outlined in class or on page 333 of the textbook. *Mark each step clearly.*

① $f(x)$ has degree 4 with leading coeff 1 so for large values of $|x|$, $f(x) \approx x^4$

② x-int: $(1,0), (-3,0), (-1,0)$

y-int: $(0-1)^2(0+3)(0+1) = 3 \Rightarrow (0,3)$

zeros:

$x=3$, mult 1 \Rightarrow cross

$x=-1$, mult 1 \Rightarrow cross

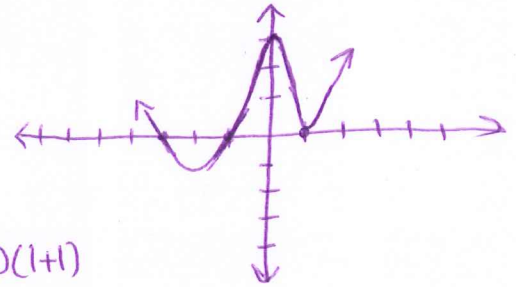
$x=1$, mult 2 \Rightarrow touch

near $x=1$:

$$f(x) \approx (x-1)^2(1+3)(1+1)$$

$$= 8(x-1)^2$$

parabola opens up



③ near $x=-3$: $f(x) \approx (-3-1)^2(x+3)(-3+1)$
 $= -32(x+3)$
 line with neg. slope

$x=-1$: $f(x) \approx (-1-1)^2(-1+3)(x+1)$
 $= 8(x+1)$
 line with pos. slope

2. (2 points) Using the Rational Zeros Theorem, find all real zeros of the following function. Also state the multiplicity of each zero.

$$g(x) = 4x^3 - 4x^2 - 7x - 2$$

$$p = \pm 1, \pm 2$$

$$q = \pm 1, \pm 2, \pm 4 \Rightarrow \frac{p}{q} = \pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$$

$$\begin{array}{r|rrrr} 2 & 4 & -4 & -7 & -2 \\ & \downarrow & 8 & 8 & 2 \\ \hline & 4 & 4 & 1 & 0 \end{array}$$

$$g(x) = (x-2)(4x^2+4x+1)$$

$$g(x) = (x-2)(2x+1)(2x+1)$$

$$g(x) = (x-2)(2x+1)^2$$

zeros: 2, mult. 1
 $-\frac{1}{2}$, mult. 2