Math 1140F - Fall 2014

Name: KEY

§6.6 — Exponential and Logarithmic Equations

Solve the following equations.

1. $-2\log_4 x = \log_4 9$

$$\log_4 x^{-2} = \log_4 9$$

$$x^{-2} = 9$$

$$\frac{1}{x^2} = 9$$

$$x^2 = \frac{1}{9}$$

$$x = \pm\frac{1}{3}$$

since

$\log_4\left(-\frac{1}{3}\right)$ is undefined,

$$\boxed{\text{Sol}^n : \left\{\frac{1}{3}\right\}}$$

2. $\log x + \log(x+15) = 2$

$$\log\left[x(x+15)\right] = 2$$

$$x(x+15) = 10^2$$

$$x^2 + 15x = 100$$

$$x^2 + 15x - 100 = 0$$

$$(x-5)(x+20) = 0$$

→ $x - 5 = 0$ or $x + 20 = 0$

$x = 5$ $x = -20$

$\log(-20)$ is undefined

$$\boxed{\text{Sol}^n : \{5\}}$$

3. $\ln x + \ln(x+2) = 4$

$$\ln\left[x(x+2)\right] = 4$$

$$x^2 + 2x = e^4$$

$$x^2 + 2x - e^4 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(-e^4)}}{2}$$

→ $x = \dfrac{-2 \pm \sqrt{4 + 4e^4}}{2}$

$= \dfrac{-2 \pm \sqrt{4(1+e^4)}}{2}$

$= \dfrac{-2 \pm 2\sqrt{1+e^4}}{2}$

$= -1 \oplus \sqrt{1+e^4}$

$$\boxed{\text{Sol}^n : \left\{-1+\sqrt{1+e^4}\right\}}$$

4. $2^{x+1} = 5^{1-2x}$

$$\log(2^{x+1}) = \log(5^{1-2x})$$

$$(x+1)\log 2 = (1-2x)\log 5$$

$$x\log 2 + \log 2 = \log 5 - 2x\log 5$$

$$x\log 2 + 2x\log 5 = \log 5 - \log 2$$

$\Rightarrow$ $x(\log 2 + 2\log 5) = \log 5 - \log 2$

$$\boxed{x = \frac{\log 5 - \log 2}{\log 2 + 2\log 5}}$$

5. $2^{2x} + 2^{x+2} - 12 = 0$

$$(2^x)^2 + 2^2 \, 2^x - 12 = 0$$

Let $w = 2^x$. Then

$$u^2 + 4w - 12 = 0$$

$$(u - 2)(u + 6) = 0$$

$\Rightarrow$ $u - 2 = 0$

$u = 2$

$2^x = 2$

$$\boxed{x = 1}$$

$w + 6 = 0$

$w = -6$

no sol^n.

6. $1.2^x = (0.5)^{-x}$

$$\log(1.2^x) = \log(0.5^{-x})$$

$$x\log(1.2) = -x\log(0.5)$$

$$0 = x\log(1.2) + x\log(0.5)$$

$$0 = x(\log(1.2) + \log(0.5))$$

$$\boxed{x = 0}$$