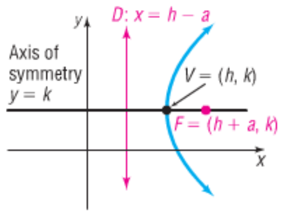
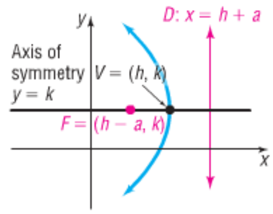
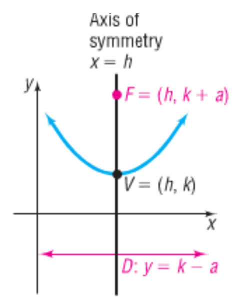
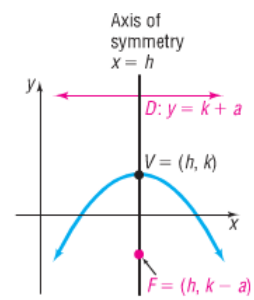
MATH 4E Fall 2012
About the Conics

1. §11.2: Parabola
   Given a parabola with vertex (h, k), we know that the parabola opens in the direction of the focus (if focus is left of vertex, parabola opens left, etc.).
   The focus and directrix (which is a line)are equal distance from the vertex. That distance is a.
   If you have $(x-h)^2 = \pm 4a(y-k)$ then the parabola is parallel to the y-axis. Similarly, if you have $(y-k)^2 = \pm 4a(x-h)$, it is parallel to the x-axis. Negatives cause the graph to flip (right to left, up to down).
   Shown below are $(y-k)^2 = \pm 4a(x-h)$ and $(z-h)^2 = \pm 4a(y-k)$ repectively.





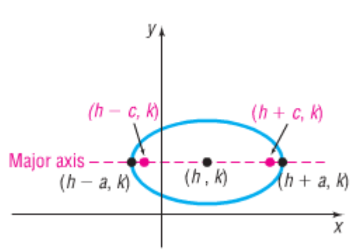


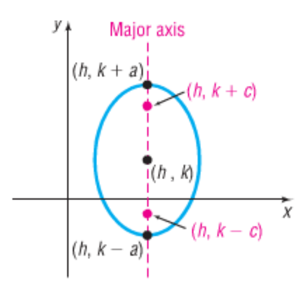


2. §11.3: Ellipse
   Given an ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

   the center of the ellipse is (h, k), and $c^2 = |a^2 - b^2|$.
   If $(y-k)^2$ has the larger denominator, then the parabola is parallel to the y-axis, and the foci and vertices are above and below the center. Vertices are distance a from the center, that is, $(h, k \pm a)$. Foci are distance c from the center, that is, $(h, k \pm c)$.
   Similarly, if $(x-h)^2$ has the larger denominator, then the parabola is parallel to the x-axis, and the foci and vertices are to the left and right of the center. Vertices are distance a from the center, that is, $(h \pm a, k)$. Foci are distance c from the center, that is, $(h \pm c, k)$.
   Shown below is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, both when $a^2 > b^2$ and when $b^2 > a^2$.

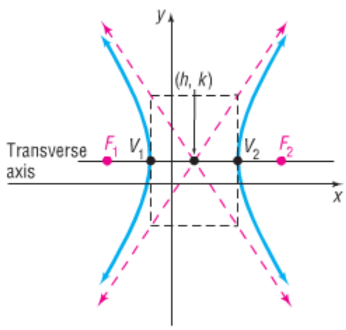


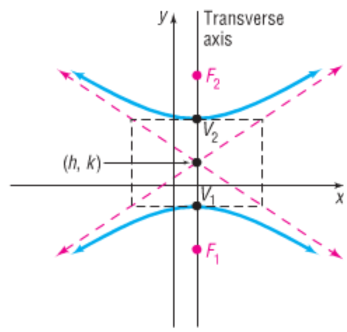


3. §11.4: Hyperbola
Given a hyperbola with center (h, k), $c^2 = a^2 + b^2$, we know that

$$\pm\frac{(x-h)^2}{a^2} \mp \frac{(y-k)^2}{b^2} = 1$$

(This says that when the $(x-h)^2$ term is positive, the $(y-k)^2$ term is negative, and vice versa.)
If the $(x-h)^2$ term is positive, then the transverse axis is parallel to the x-axis, and foci and vertices are to the left and right of the center. Foci are distance c from the center, that is $(h \pm c, k)$, and vertices are distance a from the center, that is $(h \pm a, k)$.
If the $(y-k)^2$ term is positive (equivalently, $(x-h)^2$ is negative), then the transverse axis is parallel to the y-axis, and foci and vertices are above and below the center. Foci are distance c from the center, that is $(h, k \pm c)$, and vertices are distance a from the center, that is $(h, k \pm a)$.
In both cases, asymptotes are given by $y - k = \pm\frac{b}{a}(x-h)$.
Shown below is $\pm\frac{(x-h)^2}{a^2} \mp \frac{(y-k)^2}{b^2} = 1$, both cases.





4. Notes

- When you're given a problem with conics, always start with what you know. Sketching things sometimes helps. This is all geometry, so sketching is almost necessary!
- Recognize the relationship between the points you have. Are they above/below each other? Or left/right? This will often tell you everything you need to know to form an equation.
- If you have to analyze an equation, get it into a form you recognize first. This usually means completing the square.
- Remember what you know about transformations when doing these. These problems are not as hard as they seem. They hide their information in symbols and numbers, but everything you need is right there!