All About Asymptotes

Math 1140F – Fall 2014

The following is a discussion of the content covered in §5.2 and §5.4, rational functions.

Definition 1. A **rational function** $R(x)$ is of the form

$$R(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomials. The domain of $R(x)$ will be all real numbers except for the zeros of $q(x)$. If $p(x)$ and $q(x)$ have no factors in common then $R(x)$ is *in lowest terms.* If the degree of $p(x)$ is strictly less than the degree of $q(x)$ then $R(x)$ is *proper.*

Definition 2. Let $R(x)$ denote a function.

If $|R(x)| \to \pm\infty$ as $x \to c$, then $x = c$ is a **vertical asymptote** of the graph of $R(x)$. Vertical asymptotes arise from zeros of the denominator in a rational function. Thus they can never be crossed.

If $R(x)$ approaches a fixed number L as $x \to \infty$ or $x \to -\infty$ then the line $y = L$ is a **horizontal asymptote**. If $R(x)$ approaches a line $y = ax + b$ as $x \to \infty$ or $x \to -\infty$, then $y = ax+b$ is an **oblique asymptote**. Horizontal and oblique asymptotes are end behavior trendlines. Thus, they *may* be crossed.

Locating Asymptotes

1. Vertical: If $x-r$ is a factor of the denominator polynomial $q(x)$ then $x = r$ will be a vertical asymptote.

2. Horizontal and Oblique: Let n be the degree of $p(x)$ and m the degree of $q(x)$. If $n = m$ then there will be a *horizontal* asymptote. If $n = m + 1$ then there will be an *oblique* asymptote. Further, if $R(x)$ is proper then $y = 0$ is the horizontal asymptote.

Note that you can have multiple vertical asymptotes, but at most one of horizontal and oblique asymptotes.