Math 1160K - Exam 2

Name: Key

Wednesday, October 22, 2014
Time: 50 minutes
Instructor: Brittany Cuchta

Instructions:

- Do not open the exam until I say you may.
- All cell phones and other electronic noisemaking devices must be turned off or completely silenced (i.e., not on vibrate) for the duration of the exam.
- **No calculators** are allowed on the exam.
- The exam *must* be taken in pencil. Using a pen on the exam will result in the loss of points.
- Failure to follow directions specific to a problem will result in the loss of points.
- Circle or box your final answer where appropriate. Put your final answer in the provided space when available. Failure to do so will result in points being deducted.
- Show **all** work. Full credit will only be given if work is shown which **fully and clearly** justifies your answer. I reserve the right to not grade a problem which I cannot read.
- Answers must be exact (like $\sqrt{2}$), not approximate (like 1.414), unless a problem specifically indicates otherwise.
- All final answers must be simplified unless otherwise specified. **Rationalization is not required unless otherwise specified.**
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

Page:	1	2	3	4	5	Total
Points:	17	22	32	19	10	100
Score:						

1. Find the exact value of the following expressions. If the value does not exist, state so.

(a) (2 points) $\tan^{-1}\left(\tan\frac{\pi}{3}\right) =$ $\underline{\quad \frac{\pi}{3} \quad}$

(b) (4 points) $\sin^{-1}\left(\sin\frac{5\pi}{4}\right) =$ $\underline{\quad -\frac{\pi}{4} \quad}$

$\sin\frac{5\pi}{4} = \sin\left(-\frac{\pi}{4}\right)$

(c) (4 points) $\cos\left(\csc^{-1}(-1)\right) =$ $\underline{\quad 0 \quad}$

$\cos\left(\csc^{-1}(-1)\right) = \cos\left(\sin^{-1}(-1)\right) = \cos(\pi)$

(d) (2 points) $\cos\left(\cos^{-1}1.5\right) =$ $\underline{\quad \text{DNE} \quad}$

note that $-1 \le \cos x \le 1$ so that $\cos^{-1}(1.5)$ does not exist

(e) (5 points) $\cos^{-1}\left(\tan\left(-\frac{3\pi}{4}\right)\right) =$ $\underline{\quad 0 \quad}$

$\cos^{-1}\left(\tan\left(-\frac{3\pi}{4}\right)\right) = \cos^{-1}\left(\tan\frac{3\pi}{4}\right) = \cos^{-1}(1)$

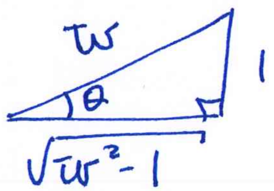
2. (6 points) Write the following as an algebraic expression in ω:

$$\tan\left(\csc^{-1}\omega\right)$$

Let $\theta = \csc^{-1} w$. Then

$\csc\theta = w$

$\theta \in QI$.



$\tan(\csc^{-1} w) = \tan\theta = \dfrac{1}{\sqrt{w^2-1}}$

Solution: $\dfrac{1}{\sqrt{w^2-1}}$

3. (6 points) Circle true or false. No partial credit will be given.

(a) True **False** : It is true that $\sin\frac{11\pi}{6} = -\frac{1}{2}$, therefore $\sin^{-1}\left(-\frac{1}{2}\right) = \frac{11\pi}{6}$.

(b) **True** False : For all x, $\tan\left(\tan^{-1} x\right) = x$.

(c) True **False** : The ranges of the inverse tangent and cotangent functions are the same.

4. (10 points) Give the general solution(s) to the following equation:

$$2\cos(2\theta) + \sqrt{3} = 0$$

$2\cos(2\theta) + \sqrt{3} = 0$

$\cos(2\theta) = -\dfrac{\sqrt{3}}{2}$

$2\theta = \dfrac{5\pi}{6} + 2k\pi \Rightarrow \theta = \dfrac{5\pi}{12} + k\pi$

$2\theta = \dfrac{7\pi}{6} + 2k\pi \Rightarrow \theta = \dfrac{7\pi}{12} + k\pi$

Solution: $\left\{\dfrac{5\pi}{12} + 2k\pi,\ \dfrac{7\pi}{12} + k\pi\right\}$

5. Solve the following equations on the interval $[0, 2\pi)$.

(a) (8 points) $2\sin^2\theta - \sin\theta = 0$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\theta = 0, \pi$ $\quad\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Solution: $\{0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi\}$

(b) (6 points) $\tan^2\alpha = 3$

$\tan^2\alpha = 3$

$\tan\alpha = \pm\sqrt{3}$

$\alpha = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Solution: $\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\}$

(c) (10 points) $\csc(3\beta) = -1$

$\csc(3\beta) = -1$

$\sin(3\beta) = -1$

$3\beta = \frac{3\pi}{2} + 2k\pi$

$\beta = \frac{\pi}{2} + \frac{2}{3}k\pi$

$\underline{k=-1}$: $\frac{3\pi}{6} - \frac{4\pi}{6} < 0$

$\underline{k=1}$: $\frac{3\pi}{6} + \frac{4\pi}{6} = \frac{10\pi}{6}$

$\underline{k=2}$: $\frac{3\pi}{6} + \frac{8\pi}{6} = \frac{11\pi}{6}$

Solution: $\{\frac{\pi}{2}, \frac{5\pi}{2}, \frac{11\pi}{6}\}$

(d) (8 points) $2\cos^2\theta - 5\cos\theta + 3 = 0$

$(2\cos\theta - 3)(\cos\theta - 1) = 0$

$2\cos\theta - 3 = 0$ or $\cos\theta - 1 = 0$

$\cos\theta = \frac{3}{2}$ $\quad\cos\theta = 1$

$\theta = 0$

no solution

Solution: $\{0\}$

6. Prove (establish) the following identities:

(a) (10 points) $\dfrac{1+\sin\theta}{1-\sin\theta} - \dfrac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta\sec\theta$

$$LHS = \frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} = \frac{(1+\sin\theta)^2-(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{1+2\sin\theta+\sin^2\theta-(1-2\sin\theta+\sin^2\theta)}{\cos^2\theta}$$

$$= \frac{4\sin\theta}{\cos^2\theta} = \frac{4\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta} = 4\tan\theta\sec\theta = RHS$$

(b) (9 points) $\dfrac{1}{\sec\theta\tan\theta} = \csc\theta + \sin(-\theta)$

$$RHS = \csc\theta + \sin(-\theta) = \csc\theta - \sin\theta$$

$$= \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta} = \cos\theta\cdot\frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sec\theta}\cdot\frac{1}{\tan\theta} = \frac{1}{\sec\theta\tan\theta} = RHS$$

7. (10 points) Find the inverse function of $f(x) = \sin(3x) + 2$, $x \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$. State the domain and range of f^{-1} in interval notation.

$$y = \sin(3x) + 2$$

$$R_f : -1 \leq \sin(3x) \leq 1$$

$$1 \leq \sin(3x) + 2 \leq 3$$

$$y - 2 = \sin(3x)$$

$$\sin^{-1}(y-2) = 3x$$

$$x = \frac{\sin^{-1}(y-2)}{3}$$

$f^{-1} =$ $\frac{1}{3}\sin^{-1}(y-2)$

Range of f^{-1}: $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

Domain of f^{-1}: $[1, 3]$