Math 6D - Exam 1

Name: KEY

Monday, March 3, 2014
Time: 50 minutes
Instructor: Brittany Whited

Instructions:

- Do not open the exam until I say you may.
- All cell phones and other electronic noisemaking devices must be turned off or completely silenced (i.e., not on vibrate) for the duration of the exam.
- **No calculators** are allowed on the exam.
- Failure to follow directions specific to a problem will result in the loss of points.
- Circle or box your final answer where appropriate.
- Show **all** work. Full credit will only be given if work is shown which **fully and clearly** justifies your answer.
- Answers must be exact (like $\sqrt{2}$), not approximate (like 1.414), unless a problem specifically indicates otherwise.
- All final answers must be simplified unless otherwise specified. **Rationalization is not required unless otherwise specified.**
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

Page:	1	2	3	4	Total
Points:	27	19	30	24	100
Score:					

1. Convert the following from radians to degrees or degrees to radians.

(a) (3 points) $75°$

$$\frac{75 \cdot \pi}{180} = \frac{15\pi}{36} = \boxed{\frac{5\pi}{12}}$$

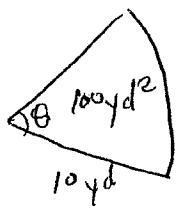
(b) (3 points) $\frac{4\pi}{15}$

$$\frac{4\pi}{15} \cdot \frac{180}{\pi} = 4 \cdot 12 = \boxed{48°}$$

2. (16 points) Complete the following table.

θ in radians	θ in degrees	$\sin\theta$	$\cos\theta$	$\tan\theta$
$\frac{\pi}{3}$	$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{5\pi}{6}$	$150°$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
$\frac{3\pi}{2}$	$270°$	-1	0	undef.
$\frac{7\pi}{4}$	$315°$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1

3. (5 points) You are asked to design a sprinkler which will cover a field of 100 yd^2 in the shape of a sector of a circle with radius 10 yd. Through what angle should the sprinkler rotate?



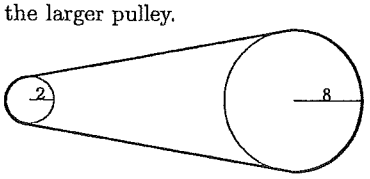
$$A = \frac{1}{2} r^2 \theta$$

$$100 = \frac{1}{2}(10)^2 \theta$$

$$100 = \frac{1}{2}(100)\theta$$

$$\boxed{\theta = 2}$$

4. (5 points) Two pulleys, one with radius 2 and one with radius 8, are connected via a belt (see diagram below). The smaller pulley rotates at a speed of 3 rev/min. Find the speed of revolution of the larger pulley.



$\frac{3 rev}{min} \cdot \frac{2\pi}{rev} = \frac{6\pi}{min} = w_1$

$V_1 = V_2$

$r_1 w_1 = r_2 w_2$

$2(6\pi) = 8 w_2$

$12\pi = 8 w_2$

$w_2 = \frac{12\pi}{8} = \frac{3\pi}{2}$

$\frac{3\pi}{2} \cdot \frac{rev}{2\pi} = \frac{3}{4} \frac{rev}{min}$

The speed of revolution of the larger pulley is $\frac{3}{4}$ rev/min

5. Solve the following expressions, giving *exact* values for each.

(a) (2 points) $\sin^2(20°) + \cos^2(20°)$

1

(identity)

(b) (3 points) $1 - \cos^2(40°) - \cos^2(50°)$

$1 - (\cos^2(40) + \cos^2(50)) = 1 - (\sin^2(90-40) + \cos^2(50))$

$= 1 - (\sin^2(50) + \cos^2(50)) = 1 - 1$

$= 0$

(c) (2 points) $\sec^2(29°) - \tan^2(29°)$

1

identity

(d) (3 points) $\cos(25°)\sin(65°) + \cos(65°)\sin(25°)$

$\cos(25)\cos(90-65) + \sin(90-65)\sin(25)$

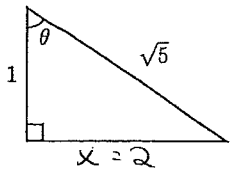
$\cos^2(25) + \sin^2(25) = 1$

(e) (4 points) $\cos^2\left(\frac{7\pi}{6}\right) + \sin^2\left(\frac{7\pi}{6}\right) + \sin^3\left(\frac{7\pi}{6}\right)$

$1 + \sin^3\left(\frac{7\pi}{6}\right) = 1 + \left(\sin\frac{7\pi}{6}\right)^3 = 1 + \left(-\frac{1}{2}\right)^3$

$= 1 - \frac{1}{8} = \frac{7}{8}$

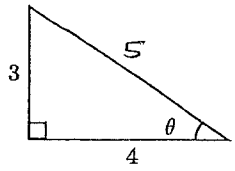
6. (24 points) Find the values for the six trigonometric functions for the following triangles.



$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{2}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	2	$\frac{\sqrt{5}}{2}$	$\sqrt{5}$	$\frac{1}{2}$

$x^2 = (\sqrt{5})^2 - 1^2 = 5 - 1$

$= 4 \Rightarrow x = 2$



$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{4}{3}$

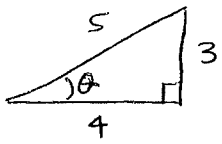
special 3-4-5 triangle!

also

$c^2 = 3^2 + 4^2 = 9 + 16 = 25$

7. (6 points) Find the exact value of the requested trigonometric function of θ given the following information:

$$\cos\theta = \frac{4}{5}, \quad 270° < \theta < 360° \quad \text{Q IV}$$



$\sin\theta =$ $-\frac{3}{5}$ $\tan\theta =$ $-\frac{3}{4}$ $\sec\theta =$ $\frac{5}{4}$

8. Graph the following function. Be sure to label key points and show at least **two full periods.**

$$y = -2\cos(2x) + 2$$

(a) (3 points) What is the amplitude of the function? $|-2| = 2$

(b) (3 points) What is the period of the function? π $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

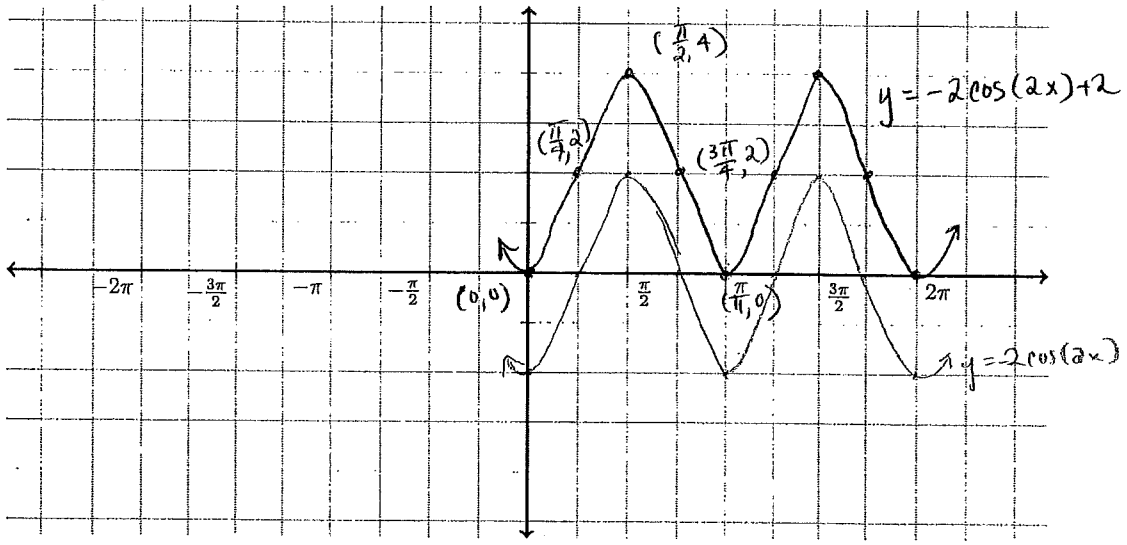
(c) (3 points) What is the phase shift of the function? 0 (none)

(d) (3 points) What is the vertical shift of the function? 2

(e) (4 points) Is the function even or odd? even

$\left[0, \frac{\pi}{4}\right], \left[\frac{\pi}{4}, \frac{\pi}{2}\right], \left[\frac{\pi}{2}, \frac{3\pi}{4}\right], \left[\frac{3\pi}{4}, \pi\right]$

(f) (4 points)



9. (4 points) The equation above is not the only equation that can be used to express this graph. Give another equation that represents the same graph.

$T = \frac{5\pi}{4} - \frac{\pi}{4} = \pi$

$\Rightarrow \omega = 2$

$\frac{\varphi}{\omega} = \frac{\pi}{4} \Rightarrow \varphi = \frac{\pi}{4} \cdot 2 = \frac{\pi}{2}$

$y = 2\sin(2x - \frac{\pi}{2}) + 2$

other answers accepted