Name: KEY

Instructor: _____

Math 6 – Trigonometry
Exam 2
March 21, 2014

Directions:

1) All cell phones and other electronic noisemaking devices must be turned off or completely silenced (i.e. not on vibrate) for the duration of the exam.

2) Show **ALL** your work! Full credit will only be given if work is shown which fully justifies your answer.

3) **No Calculators** are allowed on this exam!

4) Failure to follow directions specific to a problem will result in the loss of points.

5) Circle, box, or underline each final answer if necessary for clarity.

6) Answers must be exact (like $\sqrt{2}$), not approximate (like 1.414), unless a problem specifically indicates otherwise.

7) All final answers must be simplified unless otherwise specified. ***Rationalization is not required unless otherwise specified.***

8) This exam has 100 points possible.

9) If you need extra room, you may use the back of the previous page. However, you must indicate you are doing so by clearly writing "BPP" on the relevant problem.

10) This packet has six sheets of paper, including this cover page. Do NOT remove the staple or remove any sheet from this packet.

Page	2	3	4	5	6	Total
Possible Points	16	19	32	18	15	100
Points Earned						

1. Find the exact value of each expression. (4 points each)

a. $\cos(\cos^{-1} 1.2)$

DNE (undefined)

b. $\tan^{-1}\left(\tan\left(\frac{5\pi}{4}\right)\right)$

$\tan^{-1}(\tan\frac{5\pi}{4}) = \tan^{-1}(\tan(\frac{\pi}{4})) = \frac{\pi}{4}$

c. $\sec\left(\tan^{-1}\left(\frac{1}{2}\right)\right)$

$\sec(\tan^{-1}(\frac{1}{2}))$

let $\theta = \tan^{-1}(\frac{1}{2})$. Then

$\sec(\tan^{-1}(\frac{1}{2})) = \sec\theta = \boxed{\frac{\sqrt{5}}{2}}$

d. $\csc^{-1}(-1)$

$\csc^{-1}(-1) \Rightarrow \sin^{-1}(-1) = \frac{-\pi}{2}$

2. Write true or false for each statement. (2 points each)

false a. The domain of $y = \sin^{-1} x$ is $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$.

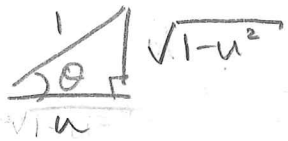
True b. $\sin(\sin^{-1} 0) = 0$ and $\cos(\cos^{-1} 0) = 0$.

true c. $y = \tan^{-1} x$ means $x = \tan y$ where $-\infty < x < \infty$ and $\frac{-\pi}{2} < y < \frac{\pi}{2}$.

3. Write the trigonometric expression as an algebraic expression in u. (5 points)

$$\csc(\cos^{-1} u)$$

let $\cos^{-1} u = \theta$. Then $\csc(\cos^{-1} u) = \csc\theta = \boxed{\frac{1}{\sqrt{1-u^2}}}$



4. Find the exact solution of each equation. (4 points each)

a. $2\cos^{-1} x = \pi$

$\cos^{-1} x = \frac{\pi}{2}$

$\boxed{x = 0}$

b. $-6\sin^{-1}(3x) = 2\pi$

$\sin^{-1}(3x) = -\frac{\pi}{3}$

$3x = \sin\left(-\frac{\pi}{3}\right)$

$\boxed{x = -\frac{\sqrt{3}}{6}}$

5. Solve the equation. Give a general formula for <u>all</u> the solutions to this equation. (8 points)

$$\sin\left(\frac{\theta}{2}\right) = -\frac{\sqrt{3}}{2}$$

$\frac{\theta}{2} = \frac{4\pi}{3} + 2k\pi \Rightarrow \theta = \frac{8\pi}{3} + 4k\pi$

$\frac{\theta}{2} = \frac{5\pi}{3} + 2k\pi \Rightarrow \theta = \frac{10\pi}{3} + 4k\pi$

6. Solve each equation on the interval $0 \le \theta \le 2\pi$. (8 points each)

a. $2\cos^2\theta + \sqrt{3}\cos\theta = 0$

$\cos\theta(2\cos\theta + \sqrt{3}) = 0$

$\cos\theta = 0$ or $2\cos\theta + \sqrt{3} = 0$

$\theta = \frac{\pi}{2} + 2k\pi$

$\theta = \frac{3\pi}{2} + 2k\pi$

$\cos\theta = -\frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{5\pi}{6} + 2k\pi$

$\theta = \frac{7\pi}{6} + 2k\pi$

$\theta \in \left\{\frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}\right\}$

b. $\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\sqrt{\frac{1}{3}}$

$\tan\theta = -\frac{1}{\sqrt{3}}$ or $\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{5\pi}{6} + 2k\pi$ $\quad$ $\theta = \frac{\pi}{6} + 2k\pi$

$\theta = \frac{11\pi}{6} + 2k\pi$ $\quad$ $\theta = \frac{7\pi}{6} + 2k\pi$

$\theta \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

c. $2\sin^2\theta - 5\sin\theta = -3$

$2\sin^2\theta - 5\sin\theta + 3 = 0$

$(2\sin\theta + 3)(\sin\theta - 1) = 0$

$2\sin\theta - 3 = 0$ OR $\sin\theta - 1 = 0$

$\sin\theta = \frac{3}{2}$ $\quad$ $\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$

DNE

7. Establish each identity. (9 points each)

a. $\tan\theta + \cot\theta = \sec\theta \csc\theta$

$$\tan\theta+\cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} = \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$$

$$= \sec\theta\csc\theta.$$

b. $9\sec^2\theta - 5\tan^2\theta = 5 + 4\sec^2\theta$

$$9\sec^2\theta - 5\tan^2\theta = \frac{9}{\cos^2\theta} - \frac{5\sin^2\theta}{\cos^2\theta}$$

$$= \frac{9-5\sin^2\theta}{\cos^2\theta}$$

$$= \frac{9-5(1-\cos^2\theta)}{\cos^2\theta}$$

$$= \frac{9-5+5\cos^2\theta}{\cos^2\theta}$$

$$= \frac{4+5\cos^2\theta}{\cos^2\theta} = \frac{4}{\cos^2\theta} + 5$$

$$= 4\sec^2\theta + 5 = \text{RHS}.$$

8. a. Find the inverse function $f^{-1}(x)$ of the following function f. (8 points)

$f(x) = \cos(x+2) + 1; \qquad -2 \le x \le \pi - 2$

$x = \cos(y+2) + 1$

$x - 1 = \cos(y+2)$

$\cos^{-1}(x-1) = y+2$

$y = \cos^{-1}(x-1) - 2$

b. Find the range of $f(x)$. Use interval notation to express your answer. (3 points)

$-1 \le \cos(x+2) \le 1$

$0 \le \cos(x+2)+1 \le 2$

$[0, 2]$

c. Find the domain of $f^{-1}(x)$. Use interval notation to express your answer. (2 points)

$-1 \le x-1 \le 1$

$0 \le x \le 2$

$[0, 2]$

d. Find the range of $f^{-1}(x)$. Use interval notation to express your answer. (2 points)

$-2 \le y \le \pi - 2$

$[-2, \pi - 2]$