Instructor's name: ____________________ Student's name: KEY

Math 6 - Exam 3

April 11, 2014

INSTRUCTIONS:

CALCULATORS MAY BE USED ON THIS EXAM

1. Be sure to print your name and your instructor's name in the space provided at the top of this page.
2. Work all problems. Show all work. Full credit will be given only if work is shown which fully justifies your answer.
3. Circle or box each final answer unless the box is provided.
4. Round all final answers to the nearest hundredth.
5. There will be sufficient space under each problem in which to show your work. If you need additional space, use the back of the page the problem is on and indicate this fact.
6. Not including the page of instructions, this exam has 4 pages. Point values are given in each problem. The total number of points is 100.
7. **Turn off and put away any electronic communications devices or cell phones, if you have these with you.**

DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO

Page	2	3	4	5	Total
Possible	32	20	29	19	100
Earned					

Product-To-Sum Formulas:

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

Sum-To-Product Formulas:

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

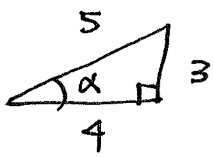
1. Establish the trigonometric identity: $\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta.$ (8 points)

$$\sin\left(\tfrac{\pi}{2}+\theta\right) = \sin\tfrac{\pi}{2}\cos\theta + \sin\theta\cos\tfrac{\pi}{2}$$
$$= (1)\cos\theta + \sin\theta\,(0)$$
$$= \cos\theta.$$

2. Find the exact value of each of the following: (8 points each)

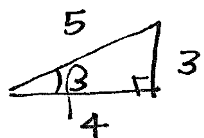
a. $\sin\left[\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(-\frac{4}{5}\right)\right]$.

$\alpha = \sin^{-1}\frac{3}{5}$



$\alpha \in QI$

$\beta = \cos^{-1}\left(-\frac{4}{5}\right)$

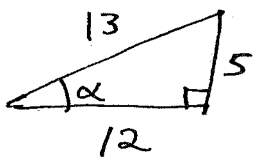


$\beta \in QII$

$$\sin\left[\sin^{-1}\tfrac{3}{5} - \cos^{-1}\left(-\tfrac{4}{5}\right)\right]$$
$$= \sin(\alpha-\beta)$$
$$= \sin\alpha\cos\beta - \sin\beta\cos\alpha$$
$$= \frac{3}{5}\left(-\frac{4}{5}\right) - \frac{3}{5}\cdot\frac{4}{5}$$
$$= \frac{-12}{25} - \frac{12}{25} = \boxed{\frac{-24}{25}}$$

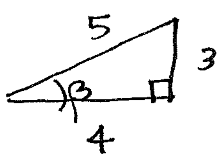
b. $\cos\left[\tan^{-1}\left(\frac{5}{12}\right) - \sin^{-1}\left(-\frac{3}{5}\right)\right]$

$\alpha = \tan^{-1}\frac{5}{12}$



$\alpha \in QI$

$\beta = \sin^{-1}\left(\frac{3}{5}\right)$



$\beta \in QIV$

$$\cos\left[\tan^{-1}\tfrac{5}{12} - \sin^{-1}\left(-\tfrac{3}{5}\right)\right]$$
$$= \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$= \frac{12}{13}\cdot\frac{4}{5} + \frac{5}{13}\cdot\frac{-3}{5}$$
$$\frac{48-15}{65} = \boxed{\frac{33}{65}}$$

3. Establish the trigonometric identity: $\cos^4\theta - \sin^4\theta = \cos 2\theta$ (8 points)

$$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$$
$$= (\cos^2\theta - \sin^2\theta)(1)$$
$$= \cos(2\theta)$$

4. Solve the following on the interval $[0, 2\pi)$. If there is no solution, state so.

(5 points each)

a. $\cos(2\theta) + 6\sin^2\theta = 4$

$$\cos(2\theta) + 6\sin^2\theta = 4$$
$$1 - 2\sin^2\theta + 6\sin^2\theta = 4$$
$$4\sin^2\theta = 3$$
$$\sin^2\theta = \frac{3}{4}$$
$$\sin\theta = \pm\frac{\sqrt{3}}{2}$$

$$\theta = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$$

b. $\cos(2\theta) = 2\cos^2\theta - 2$

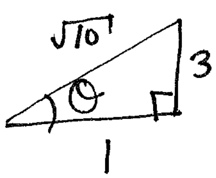
$$2\cos^2\theta - 1 = 2\cos^2\theta - 2$$
$$0 = -1 \quad \text{or} \quad -1 = -2$$

$\boxed{\text{no solution}}$

5. Find the exact value of the following:

(5 points each)

a. If $\tan\theta = -3$ and $\sin\theta < 0$, find:



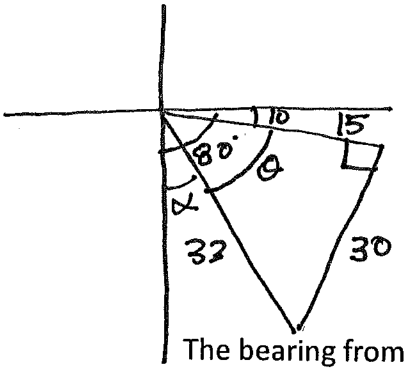
$$\frac{3\pi}{2} \le \theta \le 2\pi \Rightarrow \frac{3\pi}{4} \le \frac{\theta}{2} \le \pi$$

$$\sin\frac{\theta}{2} = \oplus\sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{1}{\sqrt{10}}}{2}} = \boxed{\sqrt{\frac{10-\sqrt{10}}{20}}}$$

b. $\tan\left(\frac{7\pi}{8}\right) = \tan\left(\frac{7\pi}{4}\cdot\frac{1}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$$-\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}} = \boxed{-\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}}$$

6. A ship leaves the port of Miami with a bearing of S80^{0}E and a speed of 15 nautical miles per hour. After 1 hour, the ship turns 90^0 toward the south. After 2 hours, maintaining the same speed, what is the bearing to the ship from the port? (10 points)



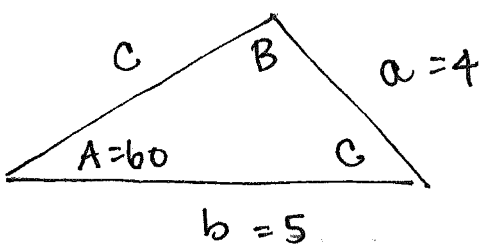
$$\tan\theta = \frac{30}{15} \Rightarrow \theta = 63.435^\circ$$

$$\alpha = 80 - \theta = 80 - 63.44 = 16.56$$

The bearing from the port to the ship is S16.56°E.

7. Two sides and an angle are given. Determine whether the given information results in one or two triangles or no triangle at all. Solve any triangle(s) that results. (9 points)

a = 4, b = 5, A = 60^0



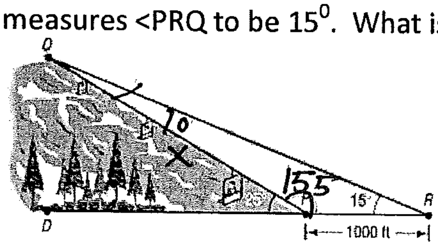
$$\frac{\sin 60}{4} = \frac{\sin B}{5}$$

$$\sin B = \frac{5\sin 60}{4} = 1.08$$

$\boxed{\text{no solution}}$

# of triangles: 0 Angle B= / , Angle C= / , c = /
Angle B= / , Angle C= / , c = / .

11. Consult the figure. To find the length of the span of a proposed ski lift from P to Q, a surveyor measures <DPQ to be 25^0 and then walks off a distance of 1000 feet to R and measures <PRQ to be 15^0. What is the distance from P to Q? (10 points)

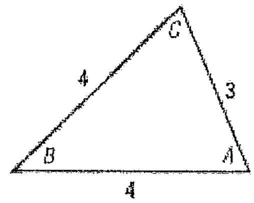


$$\frac{\sin 10}{1000} = \frac{\sin 15}{x}$$

$$x = \frac{1000\sin 15}{\sin 10}$$

$$= \boxed{1490.48ft}$$

12. Solve the triangle and find the area of this triangle. (each angle worth 3, area worth 5)



$$c^2 = a^2+b^2-2ab\cos C$$

$$4^2 = 4^2+3^2-2(4)(3)\cos C$$

$$\cos C = \frac{-9}{-24} \Rightarrow C = 67.975 = A$$

$$B = 180 - A - C$$

$$= 180 - 2(67.975)$$

$$= 44.048$$

Angle A = $67.98°$ Angle B = $44.05°$ Angle C = $67.98°$ area of ΔABC = 5.56

$$K = \sqrt{\tfrac{11}{2}\left(\tfrac{11}{2}-4\right)\left(\tfrac{11}{2}-3\right)\left(\tfrac{11}{2}-4\right)} = \sqrt{\tfrac{11}{2}\left(\tfrac{3}{2}\right)\left(\tfrac{5}{2}\right)\left(\tfrac{3}{2}\right)} = 5.562$$

$$s = \tfrac{1}{2}(4+4+3) = 5.5$$

13. Express the sum as a product of sines and/or cosines. (5 points)

$$\sin 4\theta + \sin 2\theta$$

$$\sin 4\theta + \sin 2\theta = 2\sin\left(\frac{4\theta+2\theta}{2}\right)\cos\left(\frac{4\theta-2\theta}{2}\right)$$

$$= 2\sin(3\theta)\cos(\theta)$$