Math 1160 — Summer 2016
Bonus Worksheet — §8.1, 8.2, 8.3

Name: KEY

Complete the following worksheet for up to 5 bonus points. Write your answer in the provided spaces.

1. Find the inverse function f^{-1}. State the domain and range of f^{-1} using interval or set notation.

$$f(x) = \cos(2x+1) - 1, \quad -2 \le x \le \pi - 2.$$

$y = \cos(2x+1) - 1$

$y + 1 = \cos(2x+1)$

$\cos^{-1}(y+1) = 2x+1$

$x = \dfrac{\cos^{-1}(y+1) - 1}{2}$

$-1 \le \cos(2x+1) \le 1$

$-2 \le \cos(2x+1) - 1 \le 0$

$f^{-1}(y) = \dfrac{\cos^{-1}(y+1) - 1}{2}$

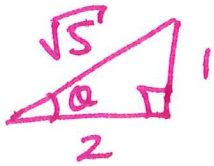
$D_{f^{-1}}$: $\{y \mid -2 \le y \le 0\} = [-2, 0]$

$R_{f^{-1}}$: $\{x \mid -2 \le x \le \pi - 2\} = [-2, \pi - 2]$

2. Find the exact values of the following:

(a) $\csc\left(\tan^{-1}\frac{1}{2}\right)$

Let $\theta = \tan^{-1}\frac{1}{2}$. Then $\tan\theta = \frac{1}{2}$ & $\theta \in QI$

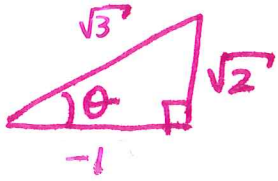


$\csc(\tan^{-1}\frac{1}{2}) = \csc\theta = \sqrt{5}$

Solution: $\sqrt{5}$

(b) $\cot\left[\cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right]$

Let $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$. Then $\cos\theta = -\frac{1}{\sqrt{3}}$ & $\theta \in QII$.



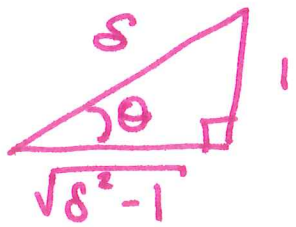
$\cot(\csc^{-1}(-\frac{1}{\sqrt{3}})) = \cot\theta = -\frac{1}{\sqrt{2}}$

Solution: $-\frac{1}{\sqrt{2}}$

3. Write the following as an algebraic expression in δ:

$$\cos\left(\csc^{-1}\delta\right)$$

Let $\theta = \csc^{-1}\delta$. Then $\csc\theta = \delta$ & we take $\theta \in QI$.



$\csc\theta = \delta \rightarrow \sin\theta = \frac{1}{\delta}$.

$\cos(\csc^{-1}\delta) = \cos\theta = \frac{\sqrt{\delta^2-1}}{\delta}$

Solution: $\frac{\sqrt{\delta^2-1}}{\delta}$

4. Find the general solution of the following:

$$3\sin(3x) - 2 = 1$$

$\sin(3x) = 1$

$3x = \frac{\pi}{2} + 2k\pi$

$x = \frac{\pi}{2} + \frac{2}{3}k\pi$

Solution: $x = \frac{\pi}{2} + \frac{2}{3}k\pi$

5. Solve the following on the interval $[0, 2\pi)$:

$$2\sin^2\theta - 5\sin\theta + 3 = 0$$

Let $x = \sin^2\theta$. Then our equation becomes

$2x^2 - 5x + 3 = 0$

$(2x-3)(x-1) = 0$

$2x - 3 = 0$ or $x - 1 = 0$

$2\sin\theta - 3 = 0$ or $\sin\theta - 1 = 0$

$\sin\theta = \frac{3}{2}$ $\boxed{\text{DNE}}$

$\sin\theta = 1$

$\theta = \frac{\pi}{2} + 2k\pi$

Solution: $x = \frac{\pi}{2}$