Written Homework - §8.4

29. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1.$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$$LHS = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$$

$$= \sec^2\theta - \tan^2\theta$$

$$= -(\tan^2\theta - \sec^2\theta)$$

$$= 1 = RHS.$$

33. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2.$

$$LHS = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$$

$$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$$

$$= 2\sin^2\theta + 2\cos^2\theta$$

$$= 2(\sin^2\theta + \cos^2\theta$$

$$= 2 = RHS.$$

39. $3\sin^2\theta + 4\cos^2\theta = 3 + \cos^2\theta$

$$LHS = 3\sin^2 + 4\cos^2\theta$$

$$= 3(1 - \cos^2\theta) + 4\cos^2\theta$$

$$= 3 - 3\cos^2\theta + 4\cos^2\theta$$

$$= 3 + \cos^2\theta = RHS.$$

41. $1 - \dfrac{\cos^2\theta}{1+\sin\theta} = \sin\theta$

$$LHS = 1 - \frac{\cos^2\theta}{1+\sin\theta}$$

$$= \frac{1+\sin\theta}{1+\sin\theta} - \frac{\cos^2\theta}{1+\sin\theta}$$

$$= \frac{1+\sin\theta-\cos^2\theta}{1+\sin\theta}$$

$$= \frac{1+\sin\theta-(1-\sin^2\theta)}{1+\sin\theta}$$

$$= \frac{\sin\theta+\sin^2\theta}{1+\sin\theta}$$

$$= \frac{\sin\theta(1+\sin\theta)}{1+\sin\theta}$$

$$= \sin\theta = RHS.$$

45. $\dfrac{\sec\theta}{\csc\theta} + \dfrac{\sin\theta}{\cos\theta} = 2\tan\theta.$

$$LHS = \frac{\sec\theta}{\csc\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}\cdot\frac{\sin\theta}{1} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta + \tan\theta$$

$$= 2\tan\theta = RHS.$$

47. $\dfrac{1+\sin\theta}{1-\sin\theta} = \dfrac{\csc\theta+1}{\csc\theta-1}$

$$LHS = \frac{1+\sin\theta}{1-\sin\theta}$$

$$= \frac{1+\sin\theta}{1-\sin\theta} \cdot \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta}}$$

$$= \frac{\frac{1}{\sin\theta} + \frac{\sin\theta}{\sin\theta}}{\frac{1}{\sin\theta} - \frac{\sin\theta}{\sin\theta}}$$

$$= \frac{\csc\theta + 1}{\csc\theta - 1} = RHS.$$

51. $\dfrac{\sin\theta}{\sin\theta-\cos\theta} = \dfrac{1}{1-\cot\theta}$

$$LHS = \frac{\sin\theta}{\sin\theta-\cos\theta}$$

$$= \frac{\sin\theta}{\sin\theta-\cos\theta} \cdot \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta}}$$

$$= \frac{\frac{\sin\theta}{\sin\theta}}{\frac{\sin\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{1}{1-\cot\theta} = RHS$$

53. $\dfrac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

$$\text{LHS} = \frac{1-\sin\theta}{1+\sin\theta}$$

$$= \frac{1-\sin\theta}{1+\sin\theta} \cdot \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{1-2\sin\theta+\sin^2\theta}{1-\sin^2\theta}$$

$$= \frac{1-2\sin\theta+\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} - 2\frac{\sin\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \sec^2\theta - 2\cdot\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta} + \tan^2\theta$$

$$= \sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta$$

$$= (\sec\theta - \tan\theta)(\sec\theta - \tan\theta)$$

$$= (\sec\theta - \tan\theta)^2 = \text{RHS}$$

57. $\tan\theta + \dfrac{\cos\theta}{1+\sin\theta} = \sec\theta$

$$LHS = \tan\theta + \frac{\cos\theta}{1+\sin\theta}$$

$$= \tan\theta + \frac{\cos\theta}{1+\sin\theta} \cdot \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \tan\theta + \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \tan\theta + \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta = RHS.$$