Chapter 5
Joint Probability Distributions and Random Samples

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Example 1a

A certain market has both and express checkout line and a superexpress checkout line. Let X_1 denote the number of customers in line at the express checkout at a particular time of day, and let X_2 denote the number of customers in line at the superexpress checkout at the same time. Suppose the joint pmf of X_1 and X_2 is as given in the accompanying table:

		x_2			
		0	1	2	3
x_1	0	0.08	0.07	0.04	0.00
	1	0.06	0.15	0.05	0.04
	2	0.05	0.04	0.10	0.06
	3	0.00	0.03	0.04	0.07
	4	0.00	0.01	0.05	0.06

- Find the probability that there is exactly one customer in each line.
- Find the probability that the number of customers in the two lines are equal.
- Let A be the event that there are at least two more customers in one line than in the other line. Express A in terms of X_1 and X_2 and calculate the probability of this event.
- What is the probability that the toal number of customers in the two lines is exactly four? At least four?

Example 2a

The joint probability distribution of the number X of cars and the number Y of buses per signal cycle at a proposed left-turn lane is displayed in the accompanying joint probability table.

$p(x,y)$		y = 0	1	2
x	0	0.025	0.015	0.010
	1	0.050	0.030	0.020
	2	0.125	0.075	0.050
	3	0.150	0.090	0.060
	4	0.100	0.060	0.040
	5	0.050	0.030	0.020

- What is the probability that there is exactly one car and one bus during a cycle? At most one car and at most one bus?
- What is the probability that there is exactly one bus during a cycle? Exactly one car?
- Suppose the left turn lane is to have a capacity of five cars and that one bus is equivalent to three cars. What is the probability of an overflow during a cycle?
- Are X and Y independent? Explain.

Example 1b

If the difference in the number of customers in line at the express checkout and the superexpress checkout is given by $X_1 - X_2$, find the expected difference. Also, find and comment on the direction and strength of the linear relationship between X_1 and X_2.

		x_2			
		0	1	2	3
x_1	0	0.08	0.07	0.04	0.00
	1	0.06	0.15	0.05	0.04
	2	0.05	0.04	0.10	0.06
	3	0.00	0.03	0.04	0.07
	4	0.00	0.01	0.05	0.06

Example 2b

Consider a small ferry that can accommodate cars and buses. The toll for cars is \$3 and the toll for buses is \$10. Let X and Y denote the number of cars and buses, respectively, carried on a single trip. Find the expected revenue from a single trip. What is the correlation coefficient of X and Y?

$p(x,y)$		y = 0	1	2
x	0	0.025	0.015	0.010
	1	0.050	0.030	0.020
	2	0.125	0.075	0.050
	3	0.150	0.090	0.060
	4	0.100	0.060	0.040
	5	0.050	0.030	0.020

Example 3

The inside diameter of a randomly selected piston ring is a random variable with mean value 12 cm and a standard deviation 0.04 cm.

- If $\bar{X}$ is the sample mean diameter for a random sample of $n = 16$ rings, where is the sampling distribution of $\bar{X}$ centered and what is the standard deviation of the $\bar{X}$ distribution?
- What about a sample of 64 rings?
- For which of these two random samples is $\bar{X}$ more likely to be within 0.01 cm of 12 cm? Explain your reasoning.
- Calculate $P(11.99 \leq \bar{X} \leq 12.01)$ when the sample size is 16.
- How likely is it that the sample mean diameter exceeds 12.01 when the sample size is 25?

Example 4

I have 45 students in my statistics class and from experience I know that the time needed to grade a randomly selected exam is a random variable with a mean time of 9 minutes and a standard deviation of 5 minutes. If I start grading at 3pm on Sunday, what is the probability I will be done in time to watch Game of Thrones, which airs at 9pm on Sunday?

Example 5

Rockwell hardness of pins of a certain type is known to have a mean value of 50 and a standard deviation of 1.2.

- If the distribution is normal, what is the probability that the sample mean hardness for a random sample of 9 pins is at least 51?
- Without assuming population normality, what is the (approximate) probability that the sample mean hardness for a random sample of 40 is at least 51?