

# Chapter 8

## Tests of a Hypothesis Based on a Single Sample

Brittany Cuchta  
STAT 3115, Spring 2015

## Example 1

A manufacturer of 40-amp fuses wants to make sure that the mean amperage at which fuses burn out is in fact 40. If the mean is lower than 40, customers will complain because the fuses require replacement too often. If the mean is higher than 40, the manufacturer might be liable for damage to an electrical system due to fuse malfunction. A sample of fuses is to be selected and inspected. If a hypothesis test were to be performed on the data, what null and alternative hypotheses would be of interest? Describe the type I and type II errors.

## Example 2

The calibration of a scale is to be checked by weighing a 10-kg test specimen 25 times. Suppose that the results of different weighings are independent of one another and that the weight on each trial is normally distributed with  $\sigma = 0.200$  kg. Let  $\mu$  denote the true average weight reading on the scale.

- ▶ What hypotheses should be tested?
- ▶ Suppose the scale is to be recalibrated if either  $\bar{x} \geq 10.1032$  or  $\bar{x} \leq 9.8969$ . What is the probability that the recalibration is carried out when it is actually unnecessary?
- ▶ What is the probability that recalibration is judged unnecessary when in fact  $\mu = 10.1$ ? When  $\mu = 9.8$ ?

## Example 3

The desired percentage of  $\text{SiO}_2$  in a certain type of aluminous cement is 5.5. To test whether the true average percentage is 5.5 for a particular production facility, 45 independently obtained samples were analyzed. The sample average was found to be 5.6 with sample standard deviation 0.3. Does this indicate conclusively that the true average percentage differs from 5.5? Carry out the analysis using a 0.01 level test. Clearly state the hypotheses, test statistic, and rejection region.

## Example 4

The melting point of each of 50 samples of a certain brand of hydrogenated vegetable oil was determined and the sample average was found to be 94.32 with standard deviation 1.20. The company aims for a melting point of 95 and they want to know if the true melting point deviates from this desired value.

- ▶ What are the hypotheses of interest?
- ▶ What is the test statistic?
- ▶ If we want a significance level of 0.01, what is the rejection region?
- ▶ What can you conclude about the true melting point of the oil?

## Type II Error Probability $\beta(\mu')$ for a Level $\alpha$ Test

Alternative Hypothesis	Probability
$H_a : \mu > \mu_0$	$\phi\left(z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$
$H_a : \mu < \mu_0$	$1 - \phi\left(-z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$
$H_a : \mu \neq \mu_0$	$\phi\left(z_{\alpha/2} + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) - \phi\left(-z_{\alpha/2} + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$

## Example 5

One method for straightening wire before coiling it into a spring is called "roller straightening." The article "The Effect of Roller and Spinner Wire Properties" (*Springs*, 1987: 27-28) reports on the tensile properties of wire. Suppose a sample of 16 wires is taken and each tested to determine tensile strength ( $\text{N}/\text{mm}^2$ ). The resulting sample mean and standard deviation are 2160 and 30, respectively.

- ▶ The mean tensile strength for springs made using spinner strengthening is  $2150 \text{ N}/\text{mm}^2$ . What hypothesis should be tested to determine whether the mean tensile strength for the roller method differs from 2150?
- ▶ Assuming the distribution is approximately normal, what conclusion would you reach for a level 0.05 test? Clearly state the test statistic and rejection region.