

## Review for Exam 2

### Identifying Discrete Distributions

Identify the most appropriate distribution, including both name and any parameters, that may be used to model each of the following random variables. The distribution will either be BINOMIAL( $n,p$ ), HYPERGEOMETRIC( $n,M,N$ ), or NEGATIVE BINOMIAL( $r,p$ ).

1.  $X$  is the number of heads in six tosses of a fair coin.

$$X \sim \text{Bin}(6, 0.5)$$

2. I will toss a fair coin until I get six heads then I will stop.  $X$  is the number of tails required until I get six heads.

$$X \sim \text{NB}(6, 0.5)$$

3. Joe tossed 80 coins, 35 of which came up heads. If we randomly pick 5 of these coin tosses,  $X$  is the number of heads in the group of 5 that we selected.

$$X \sim \text{Hyper}(5, 35, 80)$$

4. I have 10 episodes of Looney Tunes recorded. Pepe Le Pew appears in three of them. If I randomly watch 2 of the 10 episodes,  $X$  is the number of episodes I watch that Pepe appears.

$$X \sim \text{Hyper}(2, 3, 10)$$

5. Bugs Bunny dodges bullets from Yosemite Sam on a given show with probability 0.25.  $X$  is the number of shows out of the next 10 in which Bugs has to dodge bullets.

$$X \sim \text{Bin}(10, 0.25)$$

6. During a given episode, the probability that Tweety will say "I tawt I taw a puddy tat" is 0.7. (Assume he never uses the phrase twice in the same episode.)  $X$  is the number of episodes I need to watch without the phrase before I hear it for the 8<sup>th</sup> time.

$$X \sim \text{NB}(8, 0.7)$$

7. Bugs Bunny thinks he has 5 carrots, but 2 of them are really dynamite painted to look like carrots. Bugs will eat 2 of his carrots.  $X$  is the number of dynamite sticks he ingests.

$$X \sim \text{Hyper}(2, 2, 5)$$

## Practice with Calculations

8. For number (5), calculate the probability that Bugs has to dodge bullets in at least 2 of the next 10 shows.

$$P(\bar{X} \geq 2) = 1 - P(\bar{X} < 2) = 1 - \sum_{x=0}^1 \binom{10}{x} 0.25^x 0.75^{10-x}$$

$$= 1 - 0.2440$$

$$= \boxed{0.756}$$

9. For number (6), calculate the probability that the I have to watch 10 episodes before I hear the phrase for the 8<sup>th</sup> time. 10 eps  $\Rightarrow$  2 eps did not have phrase

$$P(\bar{X} = 2) = \binom{2+8-1}{8-1} 0.7^8 0.3^2 = \binom{9}{7} 0.7^8 0.3^2 = \boxed{0.1868}$$

10. For number (7), calculate the probability that both of the carrots Bugs eats are actually dynamite.

$$P(\bar{X} = 2) = \frac{\binom{2}{2} \binom{3}{3}}{\binom{5}{2}} = \frac{1}{10} = \boxed{0.1}$$

## Poisson Process Problem

11. The number of hits of a certain website follows a Poisson distribution with a mean rate of 4 per minute.

- (a) What is the probability of 5 hits in a given minute?  $\mu = \alpha \cdot t = 4 \cdot 1 = 4$

$$\bar{X} \sim \text{Poisson}(4)$$

$$P(\bar{X} = 5) = \frac{e^{-4} \cdot 4^5}{5!} = \boxed{0.1563}$$

- (b) What is the expected number of hits in two minutes?  $\mu = \alpha \cdot t = 4 \cdot 2 = 8$

$$\bar{X} \sim \text{Poisson}(8)$$

$$E(\bar{X}) = \mu = 8.$$

- (c) What is the probability that fewer than 3 hits are received in a period of 30 seconds?  $\mu = \alpha \cdot t = 4 \cdot \frac{1}{2} = 2$

$$\bar{X} \sim \text{Poisson}(2)$$

$$P(\bar{X} < 3) = \sum_{x=0}^2 \frac{e^{-2} 2^x}{x!} = 0.1353 + 0.2707 + 0.2707 = \boxed{0.6767}$$

(can also use table A.2)