Stat 215C - Exam 2

Name: KEY

Thursday, March 20, 2014
Time: 75 minutes
Instructor: Brittany Whited

Instructions:

- Do not open the exam until I say you may.
- Circle or box your final answer where appropriate.
- All work must clearly and legibly support your answer. Failure to do so will result in the loss of points.
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.
- **Please turn in your note sheet with your exam.**

Materials Allowed:

- One sheet of notes on 8.5x11 inch paper (front and back), handwritten.
- One calculator that cannot communicate with other devices. You may not share calculators during the exam.
- Cumulative distribution tables which are either provided for you or I pre-approve for use.

Page:	1	2	3	4	5	6	7	Total
Points:	18	10	15	16	10	12	19	100
Score:								

1. (6 points) Circle the correct answer. Each question is worth two points.

   (a) True or False A discrete random variable can assume only a finite number of possible values. or countably infinite

   (b) True or False : The expected value of a random variable X represents the most likely value of X. where probability is most heavily weighted

   (c) True or False: Let X be a discrete random variable with a sample space $\mathcal{S}$. Then for any $b \in \mathcal{S}$, $P(X \le b) = P(X < b)$. only true for continuous

2. (8 points) Fill in the appropriate answer in the provided blank. Each question is worth two points.

   (a) The probability mass function (pmf) $p(x)$ of a discrete random variable X is $p(0) = 0.10$, $p(1) = 0.20$, $p(2) = 0.30$, $p(3) = 0.15$, and $p(4) = 0.25$. Then the value of the cdf $F(x)$ at $x = 2$ is 0.6.
   $F(3) = P(X \le 3) = 0.1 + 0.2 + 0.3$

   (b) Let X be a discrete random variable with $V(X) = 8$. Then $V(3X + 15.6)$ is 72.
   $V(3X + 15.6) = 3^2 V(X) = 9(8)$

   (c) If X is a continuous random variable and c is any real number, then $P(X = c)$ is 0.

3. (4 points) Circle the best answer. Each question is worth two points.

   (a) Which of the following is true for a continuous random variable X with pdf $f(x)$?

      A. For any two real numbers a and b such that $a \le b$, $P(a \le X \le b) = \int_a^b f(x)\, dx$.

      B. $\int_{-\infty}^{\infty} f(x)\, dx = 1$.

      C. $f(x) \ge 0$.

      D. All of the above.

      E. None of the above.

      F. Only A and C are true, not B.

   (b) Let X be a continuous random variable with probability density function (pdf) $f(x)$ and cumulative density function (cdf) $F(x)$. Then for any two numbers a and b with a strictly less than b, which of the following is true?

      A. $P(a \le X \le b) = F(a) - F(b)$.

      B. $P(X > a) = 1 - F(a)$.

      C. $P(X > b) = F(b) - 1$.

      D. All of the above.

      E. None of the above.

      F. Only A and B are true, not C.

4. Suppose X is a discrete random variable whose cdf is given below.

$$F(x) = \begin{cases} 0 & \text{if } x < -3 \\ 0.3 & \text{if } -3 \le x < 2 \\ 0.6 & \text{if } 2 \le x < 4 \\ 0.8 & \text{if } 4 \le x < 6 \\ 1 & \text{if } x \ge 6 \end{cases}$$

(a) (2 points) Find the probability that X is greater than 4.

$P(X > 4) = 1 - P(X \le 4) = 1 - 0.8 = \boxed{0.2}$

also accepted:

$P(X \geq 4) = 1 - P(X < 4) = 1 - F(2) = 0.4$

(b) (5 points) Find the pmf of X.

X	-3	2	4	6
$f(x)$	0.3	0.3	0.2	0.2

(c) (3 points) Find $E(X)$.

$E(X) = \sum x f(x) = -3(0.3) + 2(0.3) + 4(0.2) + 6(0.2)$

$= \boxed{1.7}$

5. Assume that 1 out of every 30 people will have an adverse reaction to a new drug. The drug company takes a random sample of 55 patients who are good candidates and gives them the drug. Let Y be the number of patients that have an adverse reaction to the drug among the 55 sampled.

(a) (3 points) What kind of distribution does Y have? Be sure to give both the name and parameter values.

$$Y \sim \text{Bin}(55, \tfrac{1}{30})$$

(b) (3 points) If the company has to pay $10,000 to every patient in the sample who suffers an adverse effect, how much money should the company expect to pay?

$$E(10000X) = 10000\,E(X) = 10000(np)$$

$$= \boxed{\$18{,}333.33}$$

(c) (4 points) Find the probability that at most two patients in the sample will have an adverse reaction.

$$P(X \le 2) = \sum_{x=0}^{2} \binom{55}{x} (0.0\overline{3})^x (0.9\overline{6})^{55-x}$$

$$= 0.15496 + 0.29389 + 0.27362$$

$$= \boxed{0.72247}$$

(d) (5 points) Suppose the company had just wanted us to give a rough approximation of the amount of money they should expect to pay. What approximated distribution could you use? Is the approximation appropriate given our assumptions? Be sure to list the distribution name, parameter values, and assumptions that must be met.

poisson is the limiting distribution of binomial.

Conditions:

$n = 55 > 50$ ✓

$np = 1.8\overline{3} < 5$ ✓

∴ we may approximate with Poisson distribution where $\mu = 1.8\overline{3}$.

6. Jesse is a salesman and from past experience, he knows that 35% of people he peddles to will buy his new product. He gets \$200 from each successful sale. His boss has told him that his goal for the day is \$5000 in sales, and once he has sold enough products, he will be able to go home. **For each part of this problem, define the random variable needed for the calculation, giving both the distribution and parameter values.**

   (a) (6 points) What is the probability that the fifth person to buy will be the tenth person Jesse has talked to?

Let $Y \sim NB(5, 0.35)$.

10 people talked to, 5 will buy ⇒ 5 did not buy

$$P(Y=5) = \binom{5+5-1}{5-1}(0.35)^5(0.65)^5$$

$$= \binom{9}{4}(0.00525)(0.116029) = \boxed{0.07678}$$

   (b) (5 points) How many people should Jesse expect to talk to before he goes home?

Since Jesse needs to sell to $\frac{5000}{200} = 25$ people, let $Y \sim NB(25, 0.35)$. Then

$$E(\#\text{people}) = E(\#\text{who don't by} + \#\text{who buy}) = E(X+25)$$

$$= E(X) + 25$$

$$= \frac{25(1-0.35)}{0.35} + 25 = 71.42 \Rightarrow \boxed{71, 72 \text{ people}}$$

7. (5 points) Walt owns a car wash and he has noticed that the customers' arrival has a normal distribution with a mean time of 4.5 hours and a standard deviation of 1 hour. If the car wash opens at 8:00 am, find the probability that a customer will arrive between 11am and 2pm.

$X \sim N(4.5, 1)$

Customer arrives at 11am ⇒ 3hrs after opening

2pm ⇒ 6 hrs after opening

$$P(3 \le X \le 6) = P(3-4.5 \le Z \le 6-4.5) = P(-1.5 \le Z \le 1.5)$$

$$= \phi(1.5) - \phi(-1.5)$$

$$= 0.9332 - 0.0668$$

$$= \boxed{0.8664}$$

8. Hank the Geologist has collected 25 minerals, 10 of which are Hank's favorite, the mineral hanksite. He has asked his lab assistant Marie to take a sample of 5 for analysis. Let X be the number of hanksite specimens chosen for analysis.

(a) (3 points) What kind of distribution does X have? Be sure to list both the distribution and parameter values.

$$X \sim \text{Hypergeom}(\overset{n}{5}, \overset{M}{10}, \overset{N}{25})$$

(b) (3 points) What is the expected number of hanksite specimens among the five that Marie draws?

$$E(X) = n\frac{M}{N} = 5 \cdot \frac{10}{25} = \boxed{2}$$

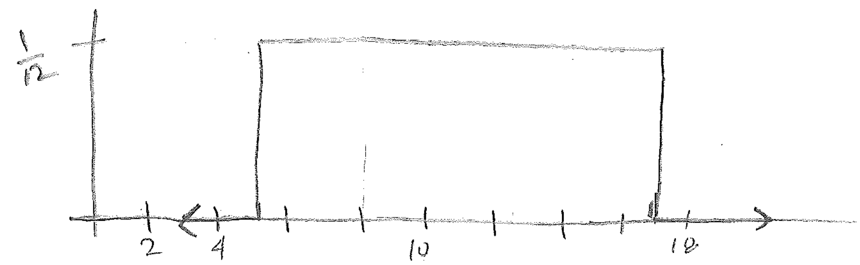
(c) (4 points) Find the probability that all five of them are hanksite.

$$P(X=5) = \frac{\binom{10}{5}\binom{15}{0}}{\binom{25}{5}} = \boxed{0.00474}$$

9. Flynn is at a breakfast bar, where customers are charged according to the amount of food they take. Let Y represent the bacon weight. Past experience shows that Y follows a uniform distribution between 5 and 17 ounces.

(a) (4 points) Write down the pdf and draw a sketch of it. Label the scales on the axes.

$$f(x) = \begin{cases} \frac{1}{12}, & 5 \le x \le 17 \\ 0, & \text{o.w.} \end{cases}$$



(b) (4 points) Find the probability that a customer will take at least 10 ounces of bacon.

$$P(X \geq 10) = \int_{10}^{17} \frac{1}{12}\,dx = \frac{17}{12} - \frac{10}{12} = \boxed{0.58\overline{3}}$$

OR

$$P(X \geq 10) = 1 - P(X < 10) = 1 - \int_{5}^{10} \frac{1}{12}\,dx = 1 - \left[\frac{10}{12} - \frac{5}{12}\right] = \boxed{0.58\overline{3}}$$

(c) (4 points) Assume that the bacon costs \$0.75 per ounce plus a base cost of \$2. Find the average customer cost for bacon.

$$E(0.75X + 2) = 0.75(E(X)) + 2$$

$$= 0.75 \int_{5}^{17} \frac{x}{12}\,dx + 2$$

$$= 0.75\left[\frac{17^2}{24} - \frac{25}{24}\right] + 2$$

$$= \boxed{\$10.25}$$

10. The reaction time (in seconds) to a certain stimulus is a continous random variable with the pdf below:

$$f(x) = \begin{cases} \frac{k}{x^2} & \text{if } 1 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) (3 points) Find the value of k that makes $f(x)$ a legitimate pdf.

$$1 = k\int_1^3 x^{-2}\,dx = k\left[-x^{-1}\right]_{x=1}^{x=3} = k\left[-\tfrac{1}{3}+1\right] = k\tfrac{2}{3} \Rightarrow \boxed{k=\tfrac{3}{2}}$$

(b) (3 points) Obtain the cdf.

$$F(x) = \int_1^x \tfrac{3}{2} y^{-2}\,dy = -\tfrac{3}{2} y^{-1}\Big|_{y=1}^{y=3} = -\tfrac{3}{2}x^{-1} + \tfrac{3}{2} \Rightarrow F(x) = \begin{cases} 1 & , x>3 \\ \tfrac{3}{2}(1-\tfrac{1}{x}) & , 1\le x\le 3 \\ 0 & , x<1 \end{cases}$$

(c) (6 points) Using either the cdf or pdf, what is the probability that the reaction time is

(a) At most 2.5 seconds?

$$P(X \le 2.5) = F(2.5) = \boxed{0.9}$$

(b) At least 1.5 seconds?

$$P(X \ge 1.5) = 1 - P(X \le 1.5) = 1 - F(1.5) = \boxed{0.5}$$

(c) Between 1.5 and 2.5 seconds?

$$P(1.5 \le X \le 2.5) = F(2.5) - F(1.5) = 0.9 - 0.5 = \boxed{0.4}$$

(d) (3 points) Find the average reaction time.

$$E(X) = \int_1^3 x f(x)\,dx = \int_1^3 \tfrac{3}{2x}\,dx = \tfrac{3}{2}\ln(x)\Big|_{x=1}^{x=3} = \boxed{1.6479}$$

(e) (4 points) Find the 75^{th} percentile of reaction time.

$$0.75 = F(x^*) = \tfrac{3}{2}\left(1 - \tfrac{1}{x^*}\right)$$

$$0.5 = 1 - \tfrac{1}{x^*}$$

$$\tfrac{1}{x^*} = 0.5$$

$$\boxed{x^* = 2}$$