

## Stat 215C - Final Exam

Name: \_\_\_\_\_

Thursday, May 8, 2014

Time: 120 minutes

Instructor: Brittany Whited

**Instructions:**

- Do not open the exam until I say you may.
- Circle or box your final answer where appropriate.
- All work must clearly and legibly support your answer. Failure to do so will result in the loss of points.
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.
- **Please turn in your note sheet with your exam.**

**Materials Allowed:**

- Two sheets of notes on 8.5x11 inch paper (front and back), handwritten.
- One calculator that cannot communicate with other devices. You may not share calculators during the exam.

[illegible]

1. True/False

- (a) (1 point) True or False : In hypothesis-testing analysis, if test statistic does not fall within the rejection region, we accept the null hypothesis.
- (b) (1 point) True or False : The probability of a type I error is often denoted by  $\alpha$  while the probability of a type II error is often denoted by  $\beta$ .

2. Fill-in-the-Blank

- (a) (4 points) The null hypothesis will be rejected if and only if the observed or computed \_\_\_\_\_ value falls within the \_\_\_\_\_.
- (b) (4 points) The standard normal random variable has a mean value of \_\_\_\_\_ and standard deviation of \_\_\_\_\_.
- (c) (2 points) When is the mean of a random sample of size  $n$  ( $n$  is small) from a normal population with mean  $\mu$ , the random variable has a probability distribution called the t-distribution with \_\_\_\_\_ degrees of freedom.

3. Multiple Choice

- (a) (2 points) A function  $p(x, y)$  of two discrete random variables  $X$  and  $Y$  can be used as a joint probability mass function provided that:
  - A.  $p(x, y) \geq 0$  for all  $x$  and  $y$ .
  - B.  $\sum_x \sum_y p(x, y) = 1$ .
  - C. Both A and B are required.
  - D. Either A or B is required.
- (b) (2 points) Which of the following are true if  $a$  and  $c$  are either both positive or both negative?
  - A.  $\text{Corr}(aX + b, cY + d) = ac\text{Corr}(X, Y)$ .
  - B.  $\text{Corr}(aX + b, cY + d) = \text{Corr}(aX, cY)$ .
  - C.  $\text{Corr}(aX + b, cY + d) = \text{Corr}(X, Y)$ .
  - D.  $\text{Corr}(aX + b, cY + d) = ac\text{Corr}(X, Y) + bd$ .
- (c) (2 points) A 99% confidence interval for the mean  $\mu$  of a normal population when the standard deviation  $\sigma$  is known is found to be (98.6, 118.4). If the confidence level is reduced to 0.95, the confidence interval for  $\mu$ 
  - A. becomes wider.
  - B. becomes narrower.
  - C. remains unchanged.
  - D. None of the above.
- (d) (2 points) In hypothesis-testing analysis, a type I error occurs only if
  - A. the null hypothesis is rejected when it is true.
  - B. the null hypothesis is rejected when it is false.
  - C. the null hypothesis is not rejected when it is false.
  - D. the null hypothesis is not rejected when it is true.
- (e) (2 points) In hypothesis-testing analysis, a type II error occurs only if
  - A. the null hypothesis is rejected when it is true.
  - B. the null hypothesis is rejected when it is false.
  - C. the null hypothesis is not rejected when it is false.
  - D. the null hypothesis is not rejected when it is true.

4. Given a sample of 40 trees, it was found that the diameter at some height has sample mean of 8.4 and sample standard deviation of 2.5.
- (a) (5 points) Find  $P(5 \leq X \leq 10)$ .
  
  
  
  
  
  
  
  
  
  
  - (b) (5 points) Suppose that the researchers continue taking samples for many years and find that the diameter of the trees follows a normal distribution with population mean 8.8 and standard deviation 2.80. Find  $P(5 \leq X \leq 10)$ .
  
  
  
  
  
  
  
  
  
  
  - (c) (7 points) Suppose that the researchers need to take back five tree samples to the lab for more measurements. They know the tree weights (in tons) follow a normal distribution with  $\mu = 2$  and  $\sigma = 1$ . Let  $X_1, \dots, X_5$  be the weights of the individual trees. Find the expected total weight the researchers will need to transfer.
5. Extensive experience with fans of a certain type used in diesel engines has suggested that the exponential distribution provides a good model for time until failure. Suppose the mean time until failure is 30,000 hours.
- (a) (4 points) What is the probability that a randomly selected fan will last at least 25,000 hours?
  
  
  
  
  
  
  
  
  
  
  - (b) (5 points) What is the probability that a randomly selected fan will last between 20,000 and 35,000 hours?
  
  
  
  
  
  
  
  
  
  
  - (c) (5 points) What is the probability that the lifetime of a fan *exceeds* the mean value by more than two standard deviations?

6. (6 points) In a sample of 507 adult Americans, it was found that only 142 correctly identified the Bill of Rights as the first ten amendments to the U.S. Constitution. Supposing this is our sample average, calculate a two-sided confidence interval using 99% confidence level for the mean amount of all U.S. adults that could give a correct description of the Bill of Rights.
7. A study of the ability of individuals to walk in a straight line gathered data on cadence (strides per second) for a sample of 20 randomly selected adults. A normal probability plot gives substantial support to the assumption that the population distribution is normal. It is found that the sample mean is 0.91 and the sample standard deviation is 0.071.
- (a) (6 points) Calculate *and interpret* a 95% confidence interval for the population mean cadence.
- (b) (6 points) Calculate *and interpret* a 95% prediction interval for the cadence of a single individual randomly selected from this population.

8. In a sample of 16 kittens it was found that the sample mean score on a cuteness scale was 8.7 with a sample standard deviation of 1.2. I claim that kittens actually have a mean cuteness of 8.9 while you disagree.

(a) (2 points) State the null and alternative hypotheses.

(b) (3 points) Determine the test statistic and compute its score.

(c) (4 points) Give the rejection region.

(d) (3 points) Make a decision regarding the null hypothesis and state your conclusion in terms of the problem.

9. An instructor has given a short test consisting of two parts. For a randomly selected student let  $X$  denote the points earned on the first part and  $Y$  denote the number of points earned on the second part. Suppose that the joint probability mass function of  $X$  and  $Y$  is given in the following table:

$x \backslash y$	0	5	10	15
0	0.01	0.02	0.04	0.05
5	0.07	0.13	0.20	0.09
10	0.01	0.21	0.14	0.03

- (a) (4 points) Find the expected value of a randomly selected student's overall score on both parts of the test.

- (b) (3 points) Are  $X$  and  $Y$  independent? Explain.

- (c) (10 points) Find  $\text{Corr}(X, Y)$  and comment on the strength and direction of the linear relationship between  $X$  and  $Y$ .

10. The article “Characterization of Room Temperature Damping in Aluminum-Indium Alloys” (*Metallurgical Trans.*, 1993: 1611-1619) suggests that Al matrix grain size ( $\mu\text{m}$ ) for an alloy consisting of 2% indium could be modeled with a normal distribution with a mean value 96 and standard deviation 14.

(a) (4 points) Find the probability that the grain size exceeds  $100\ \mu\text{m}$ .

(b) (4 points) In order to meet regulations, the grain size must be between  $95\ \mu\text{m}$  and  $100\ \mu\text{m}$ . Find the probability that a randomly selected matrix grain meets regulations.

11. (5 points) Only 1 in 1000 adults is afflicted with a rare disease for which a diagnostic test has been developed. The test is such that when an individual actually has the disease a positive result will occur 98% of the time, whereas for an individual without the disease a positive result will occur only 3% of the time. If a randomly selected individual is tested and the result is positive, what is the probability that the individual actually has the disease?

12. The three most popular options for a certain type of new car are a built-in GPS ( $A$ ), a sunroof ( $B$ ), and an automatic transmission ( $C$ ). If 40% of all purchasers request  $A$ , 55% request  $B$ , 70% request  $C$ , 63% request  $A$  or  $B$ , 77% request  $A$  or  $C$ , 80% request  $B$  or  $C$ , and 85% request  $A$ ,  $B$ , or  $C$ , determine the probability of the following events:

(a) (8 points) Draw the completed corresponding Venn diagram.

(b) (3 points) The next purchaser will request at least one of the three options.

(c) (3 points) The next purchaser will select none of the three options.

(d) (3 points) The next purchaser will request only an automatic transmission and not either of the other two options.

(e) (3 points) The next purchaser will select exactly one of these three options.



13. A friend of mine is giving a dinner party. His current wine supply includes 7 bottles of Zinfandel, 12 bottles of Merlot, and 6 bottles of Cabernet.
- (a) (3 points) If he wants to serve 3 bottles of Zinfandel and serving order is important, how many ways can he do this?
  
  
  
  
  
  
  
  
  
  
  - (b) (3 points) If 6 bottles of wine are to be randomly selected from the 35 for serving, how many ways are there to do this?
  
  
  
  
  
  
  
  
  
  
  - (c) (4 points) If 6 bottles are randomly selected, what is the probability that this results in two bottles of each variety being chosen?
  
  
  
  
  
  
  
  
  
  
  - (d) (5 points) If 6 bottles are randomly selected, what is the probability that all of them are the same variety?

14. (2 points) (a) What was your favorite section covered in this course?

(b) What was your least favorite (or most difficult) section covered in this course?

(c) What could I do to make your least favorite (or most difficult) section better?