$\boxed{3.6}$

82. Let $X \sim \text{Poisson}(\lambda = 0.2)$

(a) $P(X = 1) = \dfrac{e^{-0.2}(0.2)^1}{1!} = \boxed{0.164}$

(b) $P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - [e^{-0.2}(1+0.2)]$

$= \boxed{0.0175}$

(c) Let X_1 = # missing pulses on disk 1

X_2 = # missing pulses on disk 2

$P(X_1 = 0 \text{ and } X_2 = 0) = P(\{X_1 = 0\} \cap \{X_2 = 0\})$

by independence $= P(X_1 = 0)\, P(X_2 = 0)$

$= \dfrac{e^{-0.2}(0.2)^0}{0!} \cdot \dfrac{e^{-0.2}(0.2)^0}{0!}$

$= \boxed{0.67}$

87. Let $X \sim \text{Poisson}(\alpha = 4)$.

(a) Y = # requests in 2m period

$Y \sim \text{Poisson}(\lambda = \alpha t = 4 \cdot 2 = 8)$

$$P(Y=10) = \frac{e^{-8} 8^{10}}{10!} = \boxed{0.099}$$

(b) Z = # requests in 30min period

$Z \sim \text{Poisson}(\lambda = \frac{1}{2} \cdot 4 = 2)$

$$P(Z=0) = \frac{e^{-2}(2)^0}{0!} = e^{-2} = \boxed{0.135}$$

(c) $E(Z) = \lambda = \boxed{2}$ expected calls during lunch break.

Baseball: Let $Y \sim NB(3, 0.3)$

(a) $Y \sim NB(3, 0.3)$

(b) $E(\#\text{outs}) = E(Y) = \frac{r(1-p)}{p} = \frac{3(0.7)}{0.3} = \boxed{7}$

$E(\#\text{ at bats}) = E(\#\text{outs} + \#\text{hits})$

$= E(Y) + 3$

$= \boxed{10}$

(c) $P(Y \geq 2) = 1 - P(Y < 2) = 1 - [P(Y=0) + P(Y=1)]$

$= 1 - \left[\binom{0+3-1}{3-1}(0.3)^3(0.7)^0 + \binom{1+3-1}{3-1}(0.3)^3(0.7)^1 \right]$

$= 1 - \left[(0.3)^3 + \binom{3}{2}(0.3)^3(0.7) \right]$

$= \boxed{0.9163}$

(d) Total at bats $= 7 \Rightarrow$ outs $= 4$

$P(Y=4) = \binom{4+3-1}{3-1}(0.3)^3(0.7)^4$

$= \binom{6}{2}(0.3)^3(0.7)^4$

$= \boxed{0.0972}$

$\boxed{3.5}$

75. Let $X \sim NB(2, 0.5)$

(a). $P(X = x) = \binom{x+r-1}{r-1} p^r (1-p)^x = \binom{x+2-1}{2-1} (0.5)^2 (0.5)^x$

$= \boxed{(x+1)(0.5)^{x+2}}$

(b). four children ⇒ 2 boys

$P(X=2) = (2+1)(0.5)^{2+2} = \boxed{0.1875}$

(c) At most four children ⇒ at most 2 boys

$P(X \leq 2) = \sum_{x=0}^{2} (x+1)(0.5)^{x+2}$

$= 0.25 + 0.25 + 0.1875 = \boxed{0.6875}$

(d) $E(X) = \frac{r(1-p)}{p} = \frac{r\ 0.5}{0.5} = 2$

children = boys + girls = 4.

Airport Metal Detectors

Let X = # people among 500 who activate detector

$X \sim Bin(p=0.005, n=500)$

ⓐ Rule of Thumb: $n = 500 > 50$ ✓

$np = 500(0.005) = 2.5 < 5$ ✓

∴ we can use the Poisson approximation to the binomial.

ⓑ $P(X=5) = \binom{500}{5}(0.005)^5(0.995)^{495} = \boxed{0.066716}$

ⓒ $P(X=5) \approx \dfrac{e^{-2.5}(2.5)^5}{5!} = \boxed{0.066801}$

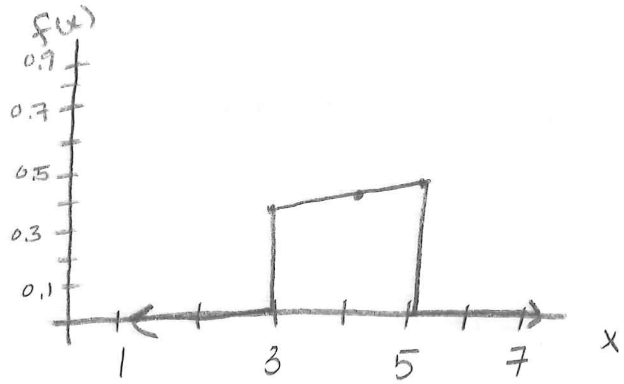
ⓓ The probabilities differ by 0.000085 (Very similar)

4.1

1.

(a) $\int_{-\infty}^{\infty} f(x)\,dx = \int_3^5 (0.075x+0.2)\,dx = \left.\frac{0.075x^2}{2} + 0.2x\right]_{x=3}^{x=5}$

$= 1.9375 - 0.9375 = 1$ ✓ valid pdf



(b) $P(X \le 4) = \int_{-\infty}^{4} f(x)\,dx = \int_3^4 (0.075x+0.2)\,dx = \left.\frac{0.075x^2}{2} + 0.2x\right|_{x=3}^{x=4}$

$= 1.4 - 0.9375 = \boxed{0.4625}$

$P(X \le 4) = P(X < 4)$ because X is a continuous RV

(c) $P(3.5 < X < 4.5) = \int_{3.5}^{4.5} (0.075x+0.2)\,dx = \left.\frac{0.075x^2}{2} + 0.2x\right|_{x=3.5}^{x=4.5}$

$= 1.659375 - 1.159375 = \boxed{0.5}$

$P(X > 4.5) = \int_{4.5}^{\infty} f(x)\,dx = \int_{4.5}^{5} (0.075x+0.2)\,dx = \left.\frac{0.075x^2}{2} + 0.2x\right|_{x=4.5}^{x=5}$

$= 1.9375 - 1.659375 = \boxed{0.278125}$

4.

(a) $\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\infty} \frac{x}{\theta^2} e^{-x^2/2\theta^2}\,dx$ let $w = \frac{-x^2}{2\theta^2}$ then $dw = \frac{-2x}{2\theta^2}dx$

$-dw = \frac{x}{\theta^2}dx$

$= -\int_0^{\infty} e^{u}\,dw = -e^{-x^2/2\theta^2}\Big|_{x=0}^{x=\infty} = 0-(-1) = 1$ ✓ valid pdf

(b) $P(X \le 200) = \int_{-\infty}^{200} f(x)\,dx = \int_0^{200}\left(\frac{x}{100^2} e^{-x^2/2(100)^2}\right)dx$

$= -e^{-x^2/2(100)^2}\Big|_{x=0}^{x=200} = -e^{-\frac{(200)^2}{2(100)^2}} + 1 = 1-e^{-2}$

$= \boxed{0.865}$

$P(X < 200) = P(X \le 200) = \boxed{0.865}$

$P(X \ge 200) = 1-P(X < 200) = 1-0.865 = \boxed{0.135}$

(c) $P(100 \le X \le 200) = \int_{100}^{200}\left(\frac{x}{100^2} e^{-x^2/2(100)^2}\right)dx = -e^{-x^2/2(100)^2}\Big|_{x=100}^{x=200}$

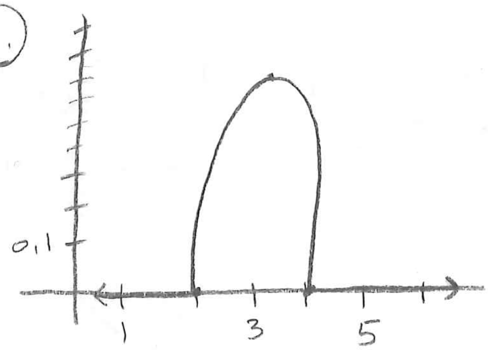
$= -e^{-(200)^2/2(100)^2} + e^{-\frac{(100)^2}{2(100)^2}} = -e^{-2} + e^{-1/2}$

$= \boxed{0.471}$

(d) $P(X \le x) = \int_{-\infty}^{x} f(y)\,dy = \int_0^{x} \frac{y}{\theta^2} e^{-y^2/2\theta^2}\,dy = -e^{-y^2/2\theta^2}\Big|_{y=0}^{y=x} = \boxed{\begin{cases} 0, & x \le 0 \\ 1-e^{-x^2/2\theta^2}, & x > 0 \end{cases}}$

6.

(a)



(b) $k\int_2^4 1-(x-3)^2\,dx = 1$

$k\left[x - \frac{(x-3)^3}{3}\right]_{x=2}^{x=4} = 1$

$k\left[4-2-\left(\frac{1}{3}+\frac{1}{3}\right)\right] = 1$

$k\left[2-\frac{2}{3}\right] = 1$

$\frac{4}{3}k = 1$

$\boxed{k = \frac{3}{4}}$

(c) $P(X > 3) = \int_3^4 \frac{3}{4}(1-(x-3)^2)\,dx$

$= \frac{3}{4}\left[x - \frac{(x-3)^3}{3}\right]_{x=2}^{x=4}$

$= \frac{3}{4}\left(4-3-\frac{1}{3}\right) = \boxed{0.5}$

(d) $P(3-0.25 \le \bar{X} \le 3+0.25) = P(2.75 \le \bar{X} \le 3.25)$

$= \int_{2.75}^{3.25} \frac{3}{4}(1-(x-3)^2)\,dx = \frac{3}{4}\left[x - \frac{(x-3)^3}{3}\right]_{x=2.75}^{x=3.25}$

$= \frac{3}{4}(0.5 - 0.01) = \boxed{0.367}$

within 0.5g

(e) $P(\bar{X} \le 3-0.5 \text{ or } \bar{X} \ge 3+0.5) = 1 - P(2.5 \le \bar{X} \le 3.5)$

$= 1 - \int_{2.5}^{3.5} \frac{3}{4}(1-(x-3)^3)\,dx = 1 - \frac{3}{4}\left[x - \frac{(x-3)^3}{3}\right]_{x=2.5}^{x=3.5} = 1 - \frac{3}{4}\left[1 - 0.08\bar{3}\right]$

$= \boxed{0.3125}$