$\boxed{4.2}$

12. (a) $P(X<0) = F(0) = \frac{1}{2} = \boxed{0.5}$

(b) $P(-1<X<1) = F(1) - F(-1)$

$= \left(\frac{1}{2} + \frac{3}{32}\left(4-\frac{1}{3}\right)\right) - \left(\frac{1}{2} + \frac{3}{32}\left(-4+\frac{1}{3}\right)\right)$

$= \frac{11}{32} + \frac{11}{32}$

$= \boxed{0.6875}$

(c) $P(0.5<X) = 1-P(X \le 0.5) = 1-F(0.5) = 1-\left[\frac{1}{2} + \frac{3}{32}\left(2 - \frac{0.5^3}{3}\right)\right]$

$= \boxed{0.316}$

(d) $f(x) = F'(x) = \frac{3}{32}\left(4 - \frac{3x^2}{3}\right) = \frac{3}{32}(4-x^2) = 0.09375(4-x^2)$

$$f(x) = \begin{cases} 0.09375(4-x^2), & -2 \le x \le 2 \\ 0, & \text{o.w.} \end{cases}$$

(e) We require x^* st $F(x^*) = 0.5$. From (a), we found $F(0) = 0.5$, thus $\boxed{\tilde{\mu} = 0.}$

14. Let X = depth of bioturbation layer in sediment

$X \sim \mathcal{U}(7.5, 20)$.

pdf: $f(x) = \frac{1}{20-7.5} = \frac{1}{12.5}$

$$f(x) = \begin{cases} \frac{1}{12.5}, & 7.5 \le x \le 20 \\ 0, & \text{o.w.} \end{cases}$$

(a) $E(X) = \frac{A+B}{2} = \frac{7.5+20}{20} = \boxed{13.75}$

$V(X) = \frac{(A-B)^2}{12} = \frac{(-12.5)^2}{12} = \boxed{13.02}$

(b) $F(x) = \int_{7.5}^{x} \frac{1}{12.5}\, dx = \frac{x-7.5}{12.5} \Rightarrow \boxed{F(x) = \begin{cases} 0, & x < 7.5 \\ \frac{x-7.5}{12.5}, & 7.5 \le x \le 20 \\ 1, & x > 20 \end{cases}}$

(c) $P(X \le 10) = F(10) = \frac{10-7.5}{12.5} = \boxed{0.2}$

$P(10 \le X \le 15) = F(15) - F(10) = \frac{15-7.5}{12.5} - 0.2 = \boxed{0.4}$

(d) $\sigma = \sqrt{13.02} = 3.608$, $\mu = 13.75$

$P(\mu - \sigma \le X \le \mu + \sigma) = P(13.75 - 3.608 \le X \le 13.75 + 3.608)$

$= P(10.142 \le X \le 17.358) = F(17.358) - F(10.142)$

$= 0.78864 - 0.21136 = \boxed{0.57728}$

$P(\mu - 2\sigma \le X \le \mu + 2\sigma) = P(6.533 \le X \le 20.967) = F(20.967) - F(6.533) = \boxed{1}$

Reaction Time

(a.)
$$F(x)=\int_{-\infty}^{x} f(y)\,dy=\int_{1}^{x}\frac{3}{2y^2}\,dy=\left.\frac{-3}{2y}\right|_{y=1}^{y=x}=-\frac{3}{2x}+\frac{3}{2}=\frac{3}{2}\left(1-\frac{1}{x}\right)$$

$$\boxed{F(x)=\begin{cases}0 & , x<1\\ \frac{3}{2}\left(1-\frac{1}{x}\right) & , 1\le x\le 3\\ 1 & , x>3\end{cases}}$$

(b.) i). $P(X\le 2.5)=F(2.5)=\frac{3}{2}\left(1-\frac{1}{2.5}\right)=\boxed{0.9}$

ii). $P(X\geq 1.5)=1-P(X<1.5)=1-F(1.5)=1-0.5=\boxed{0.5}$

iii). $P(1.5\le X\le 2.5)=F(2.5)-F(1.5)=0.9-0.5=\boxed{0.4}$

(c.)
$$E(X)=\int_{-\infty}^{\infty} x f(x)\,dx=\int_{1}^{3} x\,\frac{3}{2x^2}\,dx=\int_{1}^{3}\frac{3}{2x}\,dx$$

$$=\left.\frac{3}{2}\ln x\right|_{x=1}^{x=3}=1.6479-0=\boxed{1.6479}$$

$$E(X^2)=\int_{-\infty}^{\infty} x^2 f(x)\,dx=\int_{1}^{3} x^2\,\frac{3}{2x^2}\,dx=\left.\frac{3}{2}x\right|_{x=1}^{x=3}=\frac{9}{2}-\frac{3}{2}=3$$

$$V(X)=E(X^2)-(E(X))^2=3-(1.6479)^2=0.2844$$

$$\Rightarrow \sigma_X=\sqrt{0.2844}=\boxed{0.53326}$$

(d.) $F(x^*)=0.75$

$\frac{3}{2}\left(1-\frac{1}{x^*}\right)=0.75$

$1-\frac{1}{x^*}=0.5$

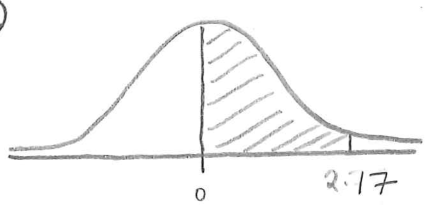
$\frac{1}{x^*}=0.5$

$\boxed{x^*=2}$

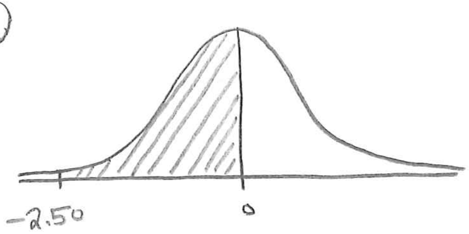
4.3

28.

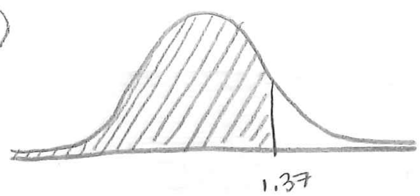
(a)



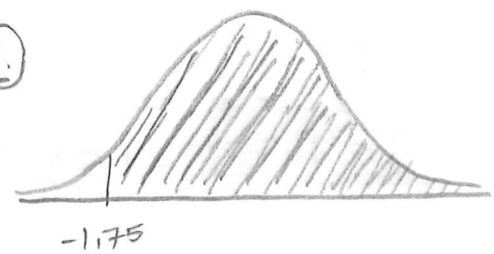
(c)



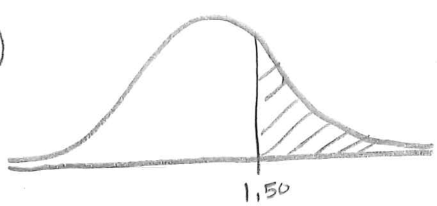
(e.)



(f.)



(i.)



36. $\bar{X}$ = droplet size when sprayed through nozzle, $\bar{X} \sim N(1050, 150)$.

(a.) $P(\bar{X} < 1500) = P\left(Z < \frac{1500-1050}{150}\right) = P(Z < 3) = \boxed{0.9987}$ Table A3

$P(\bar{X} \geq 1000) = 1 - P(\bar{X} < 1000) = 1 - P\left(Z < \frac{1000-1050}{150}\right) = 1 - P(Z < -0.\bar{3})$

from Table A.3

$= 1 - 0.3707 = \boxed{0.6293}$

(b) $P(1000 \leq \bar{X} \leq 1500) = P(0.33 \leq Z \leq 3) = \phi(3) - \phi(-0.33)$

$= \boxed{0.628}$

(c) need the 2nd percentile

$$F(x^*) = 0.02$$

$z^* = -2.05$ "inverse" reading Table A.3

$$z^* = \frac{x^* - \mu}{\sigma} = -2.05$$ unstandardize

$$\frac{x^* - 1050}{150} = -2.05$$

$$x^* = 742.5$$

only 2% of droplets are less than 742.5 μm

(d) Let X_i = droplet i's size, $i \in \{1,2,3,4,5\}$. Then we want

$P(\{X_1 \geq 1500\} \cup \{X_2 \geq 1500\} \cup \cdots \cup \{X_5 \geq 1500\})$ take complement

$= 1 - P(\{X_1 < 1500\} \cap \{X_2 < 1500\} \cap \cdots \cap \{X_5 < 1500\})$

$= 1 - P(X_1 < 1500) P(X_2 < 1500) \cdots P(X_5 < 1500)$ by independence

$= 1 - [P(X_1 < 1500)]^5$ since $X_i \sim N(1050, 150)$ $\forall i \in \{1,2,\ldots,5\}$.

$= 1 - \left[P\left(Z_1 < \frac{1500-1050}{150}\right)\right]^5 = 1 - [P(Z_1 < 3)]^5$

$= 1 - (0.9987)^5 = \boxed{0.0065}$

38. Let X_1 = machine 1 corks $\quad$ X_2 = machine 2 corks

$X_1 \sim N(3, 0.1)$ $\quad$ $X_2 \sim N(3.04, 0.02)$

To be acceptable, cork must be between 2.9, 3.1

$$P(2.9 \leq X_1 \leq 3.1)$$

$$= P\left(\frac{2.9-3}{0.1} \leq Z \leq \frac{3.1-3}{0.1}\right)$$

$$= P(-1 \leq Z \leq 1)$$

$$= \phi(1) - \phi(-1)$$

$$= 0.8413 - 0.1587 \quad \text{table A.3}$$

$$= 0.6826$$

$$P(2.9 \leq X_2 \leq 3.1)$$

$$= P\left(\frac{2.9-3.04}{0.02} \leq Z \leq \frac{3.1-3.04}{0.02}\right)$$

$$= P(-7 \leq Z \leq 3)$$

$$= \phi(3) - \phi(-7)$$

$$= 0.9987 - 0 \quad \text{table A.3}$$

$$= 0.9987$$

$\boxed{\text{∴ Machine 2 is much more likely to produce a cork of acceptable size}}$