§5.1

1.

$p(x,y)$	$y=0$	1	2
$x=0$	0.10	0.04	0.02
1	0.08	0.20	0.06
2	0.06	0.14	0.30

(a). $P(X=1, Y=1) = p(1,1) = \boxed{0.20}$

(b.) $P(X \le 1, Y \le 1) = p(0,0) + p(1,0) + p(0,1) + p(1,1)$

$= 0.10 + 0.08 + 0.04 + 0.20$

$= \boxed{0.42}$

(c.) If $A = \{X \neq 0, Y \neq 0\}$, then A is the event that at least one hose is in use at both service islands at the given time.

$P(A) = P(X \neq 0, Y \neq 0) = 1 - P(X=0, Y=0) = 1 - 0.10 = \boxed{0.9}$

(d.)

x	0	1	2
$p_X(x)$	0.16	0.34	0.5

y	0	1	2
$p_Y(y)$	0.24	0.38	0.38

$P(X \le 1) = P(X=0) + P(X=1) = p_X(0) + p_X(1) = 0.16 + 0.34 = \boxed{0.5}$

(e) Note that

$$p(0,0) = 0.10$$

but

$$p_X(0)\,p_Y(0) = 0.16(0.24) = 0.0384 \neq 0.10$$

so X, Y are not independent.

Extra: Calculate the probability that the number of hoses in use is the same at both islands.

$$P(\#\text{hoses is same}) = P(X = Y) = p(0,0) + p(1,1) + p(2,2)$$

$$= \boxed{0.60}$$

§5.1

2.

x	0	1	2
$p_X(x)$	0.5	0.3	0.2

y	0	1	2	3	4
$p_Y(y)$	0.6	0.10	0.05	0.05	0.20

(a.) Since X, Y are independent, for all (x, y)

$$p(x,y) = p_X(x)\, p_Y(y)$$

Thus the joint pmf can be given by the following table:

$p(x,y)$	y = 0	1	2	3	4
x = 0	0.3	0.05	0.025	0.025	0.10
1	0.18	0.03	0.015	0.015	0.06
2	0.12	0.02	0.01	0.01	0.04

(b.) $P(X \le 1, Y \le 1) = p(0,0) + p(0,1) + p(1,0) + p(1,1) = 0.3 + 0.05 + 0.18 + 0.03$

$= \boxed{0.56}$

Does this equal the product $P(X \le 1)P(Y \le 1)$?

$P(X \le 1)P(Y \le 1) = (p_X(0) + p_X(1))(p_Y(0) + p_Y(1))$

$= (0.5 + 0.3)(0.6 + 0.10) = 0.8(0.7)$

$= 0.56$ $\boxed{\text{YES!}}$

(c) $P(X+Y=0) = p(0,0) = \boxed{0.30}$

(d.) $P(X+Y \le 1) = p(0,0) + p(1,0) + p(0,1) = \boxed{0.53}$

$\boxed{18.}$

(a.) $p_{Y|X}(0|1) = \dfrac{p(1,0)}{p_X(1)} = \dfrac{0.08}{0.34} = \boxed{0.23529}$

$p_{Y|X}(1|1) = \dfrac{p(1,1)}{p_X(1)} = \dfrac{0.26}{0.34} = \boxed{0.588235}$

$p_{Y|X}(2|1) = \dfrac{p(1,2)}{p_X(1)} = \dfrac{0.06}{0.34} = \boxed{0.17647}$

(b.)

y	0	1	2
$p_{Y\|X}(y\|2)$	0.12	0.28	0.6

$p_{Y|X}(0|2) = \dfrac{p(2,0)}{p_X(2)} = \dfrac{0.06}{0.5}$

$p_{Y|X}(1|2) = \dfrac{p(2,1)}{p_X(2)} = \dfrac{0.14}{0.5}$

$p_{Y|X}(2|2) = \dfrac{p(2,2)}{p_X(2)} = \dfrac{0.30}{0.50}$

(c) $P(Y \le 1 \mid X=2)$

$= p_{Y|X}(0|2) + p_{Y|X}(1|2)$

$= 0.12 + 0.28$

$= \boxed{0.40}$

§5.2

22.

$p(x,y)$		y: 0	5	10	15
x	0	0.02	0.06	0.02	0.10
	5	0.04	0.15	0.20	0.10
	10	0.01	0.15	0.14	0.01

(a) $E(X+Y) = \sum_x \sum_y (x+y)\,p(x,y)$

$$= (0+5)p(0,5) + (0+10)p(0,10) + (0+15)p(0,15) + (5+0)p(5,0) + (5+5)p(5,5) + (5+10)p(5,10) + (5+15)p(5,15) + (10+0)p(10,0) + (10+5)p(10,5) + (10+10)p(10,10) + (10+15)p(10,15)$$

$$= \boxed{14.1}$$

(b) $E(\max\{X,Y\}) = \sum_x \sum_y \max\{x,y\}\,p(x,y)$

$$= \max\{0,5\}p(0,5) + \max\{0,10\}p(0,10) + \max\{0,15\}p(0,15) + \max\{5,0\}p(5,0) + \max\{5,5\}p(5,5) + \max\{5,10\}p(5,10) + \max\{5,15\}p(5,15) + \max\{10,0\}p(10,0) + \max\{10,5\}p(10,5) + \max\{10,10\}p(10,10) + \max\{10,15\}p(10,15)$$

$$= \boxed{9.6}$$

30.

(a)

x	0	5	10
$p_X(x)$	0.2	0.49	0.31

y	0	5	10	15
$p_Y(y)$	0.07	0.36	0.36	0.21

$$\mu_X = \sum_x x\, p_X(x)$$

$$= (0)(0.2) + 5(0.49) + 10(0.31)$$

$$= 5.55$$

$$\mu_Y = \sum_y y\, p_Y(y)$$

$$= 5(0.36) + 10(0.36) + 15(0.21)$$

$$= 8.55$$

$$E(XY) = \sum_x \sum_y xy\, p(x,y) = (5)(5)p(5,5) + (5)(10)p(5,10)$$
$$+ (5)(15)p(5,15) + (10)(5)p(10,5)$$
$$+ (10)(10)p(10,10) + (10)(15)p(10,15)$$
$$= 44.25$$

$$\therefore \text{Cov}(X,Y) = E(XY) - \mu_X \mu_Y = 44.25 - (5.55)(8.55) = \boxed{-3.2025}$$

(b) $E(X^2) = \sum_x x^2 p_X(x) = 5^2 p_X(5) + 10^2 p_X(10) = 43.25$

$\Rightarrow \sigma_X^2 = E(X^2) - \mu_X^2 = 43.25 - (5.55)^2 = 12.4475 \Rightarrow \sigma_X = 3.528$

$E(Y^2) = \sum_x y^2 p_Y(y) = 5^2 p_Y(5) + 10^2 p_Y(10) + 15^2 p_Y(15) = 92.25$

$\Rightarrow \sigma_Y^2 = E(Y^2) - \mu_Y^2 = 92.25 - (8.55)^2 = 151.8975 \Rightarrow \sigma_Y = 4.37579$

$$\therefore \rho = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{-3.2025}{3.528(4.376)} = \boxed{-0.2074}$$

$\therefore$ X, Y have a weak negative linear relationship