HW9

§7.3

28. (a) $t_{0.1,15} = 1.341$

(b) $t_{0.05,15} = 1.753$

(c) $t_{0.05,25} = 1.708$

35. $n = 15\ (< 30)$, $\bar{x} = 25.0$, $s = 3.5$ $\Rightarrow$ use t!

(a.) $\alpha = 0.05$, 95% CI

$$\bar{x} \pm t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}} = 25.0 \pm 2.131 \cdot \frac{3.5}{\sqrt{15}} = \boxed{(23.1, 26.9)}$$

(b.) $\alpha = 0.05$, 95% PI

$$\bar{x} \pm t_{\frac{\alpha}{2},n-1} \cdot s\sqrt{1+\frac{1}{n}} = 25.0 \pm 2.131(3.5)\sqrt{1+\frac{1}{15}} = \boxed{(17.3, 32.7)}$$

The prediction interval is much wider than the confidence interval.

HW9

37. $n = 20 (< 30)$, $\bar{x} = 0.9255$, $s = 0.0809$

$\Rightarrow$ use t!

(a.) $\alpha = 0.05$, 95% CI

$$\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \cdot s/\sqrt{n} = 0.9255 \pm 2.093 \cdot \frac{0.0809}{\sqrt{20}}$$

$$= \boxed{(0.8876, 0.9634)}$$

(b.) $\alpha = 0.05$, 95% PI

$$\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \cdot s\sqrt{1+\frac{1}{n}} = 0.9255 \pm 2.093\,(0.0809)\sqrt{1+\frac{1}{20}}$$

$$= \boxed{(0.7520, 1.0990)}$$

50. $n = 5$

95% CI $= (229.764, 233.504)$

$\bar{x}$ must be in the center. So, $\bar{x} = \dfrac{229.764 + 233.504}{2} = 231.634$

Use width for s: $w = 2\, t_{0.025,4} \left(\frac{s}{\sqrt{n}}\right)$

$$3.74 = 2(2.776)\frac{s}{\sqrt{5}} \Rightarrow s = \frac{\sqrt{5}\,(3.74)}{2(2.776)} = 1.5063$$

Thus, a 99% CI is

$$\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}} = 231.634 \pm 4.604 \cdot \frac{1.5063}{\sqrt{5}} = \boxed{(228.53, 234.74)}$$

52.

HW9

$n=5$, $\bar{x}=24.3$, $s=4.1$

(a.) $\alpha=0.05$, 95% CI

$$\bar{x} \pm t_{\frac{\alpha}{2},n-1} \frac{s}{\sqrt{n}} = 24.3 \pm 2.776 \cdot \frac{4.1}{\sqrt{5}} = \boxed{(19.21, 29.39)}$$

We are 95% certain that the true average arsenic concentration in all such water specimens lies within this interval.

(c.) $\alpha=0.05$, 95% PI

$$\bar{x} \pm t_{\frac{\alpha}{2},n-1} \cdot s\sqrt{1+\frac{1}{n}} = 24.3 \pm 2.776(4.1)\sqrt{1+\frac{1}{5}}$$

$$= \boxed{(11.83, 36.77)}$$