Section 2.5

77: Let S be the event that an individual rivet does not need reworked. Then

a). $$P(\text{ream fails}) = 0.2$$
$$1 - P(\text{ream succeed}) = 0.2$$
$$P(\text{ream succeed}) = 0.8$$
$$P(\underbrace{S \cap S \cap \cdots \cap S}_{25 \text{ times}}) = 0.8$$

$$P(S)P(S)P(S)\cdots P(S) = 0.8 \quad \text{by independence}$$
$$(P(S))^{25} = 0.8$$
$$P(S) = (0.8)^{\frac{1}{25}} = 0.9911\cdots$$

$$\therefore P(F) = 1 - P(S) = 1 - 0.9911\cdots = \boxed{0.0089}$$

b). Similar to (a), but we now require that $P(\text{ream fails}) = 0.1$. Thus:

$$P(S)^{25} = 0.9$$
$$P(S) = (0.9)^{\frac{1}{25}}$$
$$P(F) = 1 - (0.9)^{\frac{1}{25}} = \boxed{0.004}$$

Section 3.1

10: a). possible values: $\{0,1,2,3,4,5,6,7,8,9,10\} = T$

b). possible values: $\{-4,-3,-2,-1,0,1,2,3,4,5,6\} = X$

c). possible values: $\{0,1,2,3,4,5,6\} = U$

d). possible values: $\{0,1,2\} = Z_1$

Section 3.2

12: a). In order to accomodate all who show up, we can have no more than 50 show up:

$$P(Y \leq 50) = 0.05 + 0.1 + 0.12 + 0.14 + 0.25 + 0.17$$
$$= \boxed{0.83}$$

b). For not all to be accomodated, we need greater than 50. Using (a):

$$P(Y > 50) = 1 - P(Y \leq 50) = 1 - 0.83 = \boxed{0.17}$$

c). If you are the person on first standby, you need 49 people (or less) to show up. Thus:

$$P(Y \leq 49) = 0.05 + 0.1 + 0.12 + 0.14 + 0.25$$
$$= \boxed{0.66}$$

Similarly, if you are the third person on standby, you need no more than 47 people to show up:

$$P(Y \leq 47) = 0.05 + 0.1 + 0.12 = \boxed{0.27}$$

14: a). We require k such that $\sum p(y) = 1$. To this end,

$$\sum p(y) = \sum_{y=1}^{5} ky \overset{\text{want}}{\equiv} 1$$

$$k(1+2+3+4+5) = 1$$

$$k\,15 = 1$$

$$\boxed{k = \frac{1}{15}}$$

b). $P(Y \leq 3) = P(1) + P(2) + P(3)$

$$= \frac{1}{15} + \frac{2}{15} + \frac{3}{15}$$

$$= \boxed{0.4}$$

c). $P(2 \leq Y \leq 4) = p(2) + p(3) + p(4)$

$$= \frac{2}{15} + \frac{3}{15} + \frac{4}{15}$$

$$= \boxed{0.6}$$

d). Check that $p(y) = \frac{y^2}{50}$ is a valid pmf:

$$\sum_{y=1}^{5} \frac{y^2}{50} = 50(1+4+9+16+25) = 1.1 > 1$$

$\therefore p(y) = \frac{y^2}{50}$ is <u>not</u> a valid pmf

$\boxed{\text{Extra problems}}$

1: a). We require c such that $\sum p(x) = 1$.

$$\sum_{x=0}^{7} p(x) = 1$$

$$0.1 + 0.15 + 0.15 + 0.2 + 0.1 + 0.1 + 0.15 + C = 1$$

$$0.95 + C = 1$$

$$\boxed{C = 0.05}$$

b). $F(0) = P(X \leq 0) = p(0) = 0.1$

$F(1) = P(X \leq 1) = p(0) + p(1) = 0.1 + 0.15 = 0.25$

$F(2) = P(X \leq 2) = p(0) + p(1) + p(2) = 0.1 + 0.15 + 0.15 = 0.4$

Continuing in this way,

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.1, & 0 \leq x < 1 \\ 0.25, & 1 \leq x < 2 \\ 0.4, & 2 \leq x < 3 \\ 0.6, & 3 \leq x < 4 \\ 0.7, & 4 \leq x < 5 \\ 0.8, & 5 \leq x < 6 \\ 0.95, & 6 \leq x < 7 \\ 1, & x \geq 7 \end{cases}$$

c). $P(\bar{X} \geq 4) = 1 - P(\bar{X} < 4)$
$= 1 - P(\bar{X} \leq 3)$
$= 1 - 0.6$
$= \boxed{0.4}$

$P(\bar{X} = 4) = p(4) = \boxed{0.1}$

$P(2 \leq \bar{X} \leq 5) = F(5) - F(1) = 0.8 - 0.25 = \boxed{0.55}$

$P(\bar{X} \geq 0) = \boxed{1}$

* Note that all probabilities in (c) can be calculated using pmf or cdf.

2. a). The pmf is defined at jump points

$p(\bar{X} = 1) = F(1) - F(0) = 0.3 - 0 = 0.3$
$P(\bar{X} = 3) = F(3) - F(2) = 0.4 - 0.3 = 0.1$
$P(\bar{X} = 4) = F(4) - F(3) = 0.45 - 0.4 = 0.05$

Continuing in this way:

x	1	3	4	6	12
p(x)	0.3	0.1	0.05	0.15	0.4

b). $P(\bar{X} = 3) = p(3) = \boxed{0.1}$

$P(\bar{X} > 3) = p(4) + p(6) + p(12) = 0.05 + 0.15 + 0.4 = \boxed{0.6}$

$P(3 \leq \bar{X} \leq 6) = p(3) + p(4) + p(6) = 0.1 + 0.05 + 0.15 = \boxed{0.3}$