Section 3.3

32. X = amt of storage space purchased by the next customer.

x	13.5	15.9	19.1
$p(x)$	0.2	0.5	0.3

a. $E(X) = \sum_x x\,p(x) = 13.5(0.2) + 15.9(0.5) + 19.1(0.3)$
$= \boxed{16.38}$

$E(X^2) = \sum_x x^2 p(x) = (13.5)^2(0.2) + (15.9)^2(0.5) + (19.1)^2(0.3)$
$= \boxed{272.298}$

$V(X) = E(X^2) - [E(X)]^2 = 272.298 - (16.38)^2$
$= \boxed{3.99}$

b. $E(25X - 8.5) = 25E(X) - 8.5 = 25(16.38) - 8.5 = \boxed{\$401}$

c. $V(25X - 8.5) = (25)^2 V(X) = 625(3.99) = \boxed{\$2493.75}$

d. $E(X - 0.01X^2) = E(X) - 0.01E(X^2) = 16.38 - 0.01(272.298)$
$= \boxed{13.66}$

Section 3.4

60. Let X be the number of passenger cars. Then
$$X \sim Bin(25, 0.6).$$

$$Toll = \text{passenger toll} + \text{other toll}$$
$$= x + 2.50(25 - x) = 62.5 - 1.5x$$

$$E(toll) = E(62.5 - 1.5X) = 62.5 - 1.5E(X)$$
$$= 62.5 - 1.5(25)(0.6)$$
$$= \boxed{\$40}$$

Battery Voltages

Let X be the number of batteries with unacceptable voltages in our sample. Then $X \sim Bin(32, 0.05)$.

a. $X \sim Bin(32, 0.05)$.

b. $E(X) = np = 32(0.05) = \boxed{1.6}$

$V(X) = npq = 32(0.05)(0.95) = \boxed{1.52}$

c. $P(none) = P(X = 0) = \binom{32}{0} 0.05^0 \, 0.95^{32} = \boxed{0.194}$

d. $P(\text{at least one}) = 1 - P(none) = 1 - 0.194 = \boxed{0.806}$

Section 3.5

76. The only options for X are 3, 4, and 5.

x	3	4	5
$p(x)$	0.25	0.375	0.375

For $P(X=3)$, if they have 3 boys then the probability is $(0.5)^3 = 0.125$. But they could also have three girls! So

$$P(X=3) = P(3 \text{ boys or } 3 \text{ girls}) = 0.5^3 + 0.5^3 = 0.25$$

For $P(X=4)$, suppose they have 3 boys (the girls' case will be analogous).

$$\_\ \_\ \_\ B$$

The last child must be a boy, leaving 3 spots to fill with 2 other boys. There are $\binom{3}{2}$ ways to do this. Further, there are 4 children total for a probability of 0.5^4. So

$$P(X=4) = 2\cdot\binom{3}{2}0.5^4 = 0.375$$

For $P(X=5)$, the process is equivalent to $X=4$ or note that $p(5) = 1 - p(3) - p(4)$.

Baseball

Let Y = number of strike outs before 3^{rd} hit.

Note that the problem states that either the player gets a hit or is out, so we are not using the 3 strikes to be out rule.

a. $Y \sim NB(3, 0.4)$.

b. $E(Y) = \frac{r(1-p)}{p} = \frac{3(0.6)}{0.4} = \boxed{4.5}$

$E(\text{bats}) = E(\text{miss} + \text{hits}) = E(Y+3) = 4.5+3 = \boxed{7.5}$

c. $P(Y \geq 2) = 1 - P(Y<2) = 1 - (P(Y=0) + P(Y=1))$

$= 1 - \left[\binom{2}{2} 0.4^3 0.6^0 + \binom{3}{2} 0.4^3 0.6^1\right]$

$= 1 - 0.1792$

$= \boxed{0.8208.}$

d. Total at bats $= 7 \Rightarrow$ misses $= 4$

$P(Y=4) = \binom{6}{2} 0.4^3 0.6^4 = \boxed{0.124}$

Section 3.6

82. $X \sim \text{Poisson}(0.2)$

a. $P(X=1) = \dfrac{e^{-0.2}(0.2)^1}{1} = \boxed{0.1637}$

b. $P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.982 = \boxed{0.018}$

from Table A.2

<u>OR</u> $1 - P(X \leq 1) = 1 - [P(X=0) + P(X=1)] = 1 - [e^{-0.2} + 0.2e^{-0.2}] = \boxed{0.018}$

c. P(first and second no missing) = P(first) P(second)

$= P(X_1 = 0)\, P(X_2 = 0)$

$= 0.819(0.819)$

$= \boxed{0.670}$

87. $\alpha = 4$ per hour

a. $\mu = \alpha t = 8$

$P(X=10) = \dfrac{e^{-8}8^{10}}{10!} = \boxed{0.099}$

b. $\mu = \alpha t = 2$

$P(X=0) = \dfrac{e^{-2}2^{0}}{0!} = \boxed{0.135}$

c. $E(X) = \alpha t = \boxed{2}$

Airport Metal Detectors

a. We can approximate using Poisson if $n > 50$ and $np < 5$. Here, $n = 600 > 50$ and $np = 3 < 5$, so

$\boxed{\text{we may approximate using Poisson!}}$

b. $X \sim Bin(600, 0.005)$

$$P(X=5) = \binom{600}{5} 0.005^{5}\, 0.995^{595} = \boxed{0.100902}$$

c. $$P(X=5) \approx \frac{e^{-3} 3^{5}}{5!} = \boxed{0.100819}$$

d. The probabilities differ by 0.0000836 (very similar).