Section 4.4

61. $X \sim Exp(\lambda)$. We are given that $\mu = 2.725$. Thus $\lambda = 0.36697$

a. $P(X \geq 2) = 1 - P(X < 2) = 1 - F(2; \lambda)$

$$= 1 - [1 - e^{-\lambda \cdot 2}]$$

$$= e^{-2\lambda}$$

$$= \boxed{0.4800}$$

$P(X \leq 3) = F(3; \lambda) = 1 - e^{-\lambda \cdot 3} = 1 - 0.3325\ldots = \boxed{0.6674}$

$P(2 \leq X \leq 3) = F(3; \lambda) - F(2; \lambda) = 0.6674 - 0.5199 = \boxed{0.1474}$

b. $P(X > \mu + 2\sigma) = P(X > 2.725 + 2(2.725))$

$$= P(X > 8.175)$$

$$= 1 - P(X \leq 8.175)$$

$$= 1 - F(8.175; \lambda)$$

$$= e^{-8.175\lambda}$$

$$= \boxed{0.0498}$$

$P(X < \mu - \sigma) = P(X < 0) = \boxed{0.}$

63.

a. For calls of a shorter duration, plan 1 would be better. For longer calls, plan 2 will be better.

b. $h_1(\bar{X}) = 10\bar{X}$

$$h_2(\bar{X}) = \begin{cases} 99, & \bar{X} \le 20 \\ 99 + 10(\bar{X} - 20), & \bar{X} > 20 \end{cases}$$

Then,

$$E(h_1(\bar{X})) = E(10\bar{X}) = 10E(\bar{X}) = 10\mu$$

$$\begin{aligned} E(h_2(\bar{X})) &= 99 + 10E(\bar{X} - 20) \\ &= 99 + 10\int_{20}^{\infty}(x-20)\lambda e^{-\lambda x}\,dx \\ &= 99 + \tfrac{10}{\lambda}e^{-20\lambda} \\ &= 99 + 10\mu e^{-20/\mu} \end{aligned}$$

use integration by parts!

When $\mu = 10$: $E(h_1(\bar{X})) = 100 = \$100$

$E(h_2(\bar{X})) = 99 + 10(10)e^{-2} \approx \$1.13$

$\mu = 15$: $E(h_1(\bar{X})) = 150 = \$1.50$

$E(h_2(\bar{X})) = 99 + 150e^{-\frac{4}{3}} \approx \$1.39$

As expected, for shorter calls plan 1 is better; for longer calls plan 2 is better.

Section 5.1

1. a. $P(X=1 \text{ and } Y=1) = p(1,1) = \boxed{0.2}$

b. $P(X \le 1 \text{ and } Y \le 1) = p(0,0) + p(0,1) + p(1,0) + p(1,1)$
$= \boxed{0.2}$

c. The event $\{X \ne 0 \text{ and } Y \ne 0\}$ is the event that each island has at least one pump in use.

$P(X \ne 0 \text{ and } Y \ne 0) = 1 - P(X=0 \text{ and } Y=0) = 1 - 0.1 = \boxed{0.9}$

d.

x	0	1	2
$p_X(x)$	0.16	0.34	0.5

y	0	1	2
$p_Y(y)$	0.24	0.38	0.38

$P(X \le 1) = p_X(0) + p_X(1) = 0.16 + 0.34 = \boxed{0.5}$

e. X and Y are <u>not</u> independent. Consider

$p_X(0)\, p_Y(0) = 0.16 \cdot 0.24 = 0.0384 \ne 0.10 = p(0,0).$

Extra: $P(X=Y) = p(0,0) + p(1,1) + p(2,2) = \boxed{0.6}$

2. X = number of headlights needing adjustment
Y = number of tires that are defective

a.

$p(x,y)$		y = 0	1	2	3	4
x	0	0.3	0.05	0.025	0.025	0.1
	1	0.18	0.03	0.015	0.015	0.06
	2	0.12	0.02	0.01	0.01	0.04

b. $P(X \le 1, Y \le 1) = p(0,0) + p(0,1) + p(1,0) + p(1,1) = \boxed{0.56}$

$P(X \le 1) \cdot P(Y \le 1) = (p_X(0) + p_X(1)) \cdot (p_Y(0) + p_Y(1)) = 0.8 \cdot 0.7 = 0.56$

$\therefore P(X \le 1, Y \le 1) = P(X \le 1)\, P(Y \le 1).$

c. $P(X + Y = 0) = P(X = 0, Y = 0) = p(0,0) = \boxed{0.3}$

d. $P(X + Y \le 1) = p(0,0) + p(1,0) + p(0,1) = \boxed{0.53}$

8. Supplier 1 : 8

2 : 10

3 : 12

we choose six components at a time.

X = number of supplier 1 picked

Y = number of supplier 2 picked

a. $p(3,2) = P(3$ from S.1 and 2 from S.2$)$

$$= \frac{\binom{8}{3}\binom{10}{2}\binom{12}{1}}{\binom{30}{6}} = \boxed{0.0509}$$

b.

$$p(x,y) \begin{cases} \dfrac{\binom{8}{x}\binom{10}{y}\binom{12}{6-(x+y)}}{\binom{30}{6}} & , \; x, y \in \mathbb{Z}^+ \text{ such that } 0 \leq x+y \leq 6 \\ 0 & , \; \text{o.w.} \end{cases}$$

18.

a. $p_{Y|X}(0|1) = \dfrac{p(1,0)}{p_X(1)} = \dfrac{0.08}{0.34} = \boxed{0.2353}$

$p_{Y|X}(1|1) = \dfrac{p(1,1)}{p_X(1)} = \dfrac{0.2}{0.34} = \boxed{0.5882}$

$p_{Y|X}(2|1) = \dfrac{p(1,2)}{p_X(1)} = \dfrac{0.06}{0.34} = \boxed{0.1765}$

b. We want $p_{Y|X}(y|2)$. To obtain this, divide each entry in the $x=2$ row by $p_X(2) = 0.5$.

y	0	1	2
$p_{Y\|X}(y\|2)$	0.12	0.28	0.60

c. $P(Y \le 1 \mid X = 2) = 0.12 + 0.28 = \boxed{0.40}$

d. $p_{X|Y}(x|2)$ results from dividing each entry in the $y=2$ column by $p_Y(2) = 0.38$.

x	0	1	2
$p_{X\|Y}(x\|2)$	0.0526	0.1579	0.7895

Section 5.2

22.

a. $E(X+Y) = \sum_x \sum_y (x+y)\,p(x,y) = \sum_x x \sum_y p(x,y) + \sum_y y \sum_x p(x,y)$

$= \boxed{14.10}$

b. $E(\max\{X,Y\}) = \sum_x \sum_y \max\{x,y\}\,p(x,y) = \boxed{9.60}$

30.

a.

x	0	5	10
$p_X(x)$	0.2	0.49	0.31

y	0	5	10	15
$p_Y(y)$	0.07	0.36	0.36	0.21

$\mu_X = \sum x\, p_X(x) = 5(0.49) + 10(0.31) = 5.55$

$\mu_Y = \sum y\, p_Y(y) = 5(0.36) + 10(0.36) + 15(0.21) = 8.55$

$E(XY) = \sum xy\,p(x,y) = 44.25$

$\therefore Cov(X,Y) = E(XY) - \mu_X \mu_Y = 44.25 - 5.55(8.55) = \boxed{-3.20}$

b. $E(X^2) = \sum x^2 p_X(x) = 43.25 \Rightarrow \sigma_X^2 = 43.25 - (5.55)^2 = 12.4475$

$E(Y^2) = \sum y^2 p_Y(x) = 92.25 \Rightarrow \sigma_Y^2 = 92.25 - (8.55)^2 = 19.1475$

$\therefore \rho = \dfrac{Cov(X,Y)}{\sigma_X \sigma_Y} = \dfrac{-3.20}{\sqrt{12.45 \cdot 19.15}} = \boxed{-0.207}$

$\therefore$ there is a weak negative linear relationship between X and Y.