Section 5.4

50. $\mu = 10\,000$
$\sigma = 500$

a). $n = 40$

Since $n > 30$, we may use CLT to approximate probabilities using the normal distribution. Then

$$\mu_{\bar{x}} = \mu = 10000$$
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 79.057$$

$$P(9900 \le \bar{X} \le 10\,200) = P\left(\frac{9900 - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \le Z \le \frac{10200 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$$

$$= P(-1.26 \le Z \le 2.53)$$
$$= \phi(2.53) - \phi(-1.26)$$
$$= 0.9941 - 0.1038$$
$$= \boxed{0.8903}$$

b). If the sample size had been 15 instead of 40 we would not have been able to use CLT and thus the distribution of $\bar{x}$ would have been unknown. Thus, we would not have been able to find the requested information.

54. $\mu = 2.65$
$\sigma = 0.85$

population is normally distributed

a). Since the population is normal, any random sample is also normal. Thus for $n = 25$

$$E(\bar{X}) = 2.65$$
$$SD(\bar{X}) = 0.17$$

$$P(\bar{X} \le 3) = P\left(Z \le \frac{3-2.65}{0.17}\right) = P(Z \le 2.06) = \boxed{0.9803}$$

$$P(2.65 \le \bar{X} \le 3) = P\left(\frac{2.65-2.65}{0.17} \le \bar{X} \le \frac{3-2.65}{0.17}\right)$$
$$= P(0 \le \bar{x} \le 2.06)$$
$$= \phi(2.06) - \phi(0)$$
$$= 0.9803 - 0.5$$
$$= \boxed{0.4803}$$

b). We require that
$$P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \le \frac{3-\mu}{\sigma/\sqrt{n}}\right) = 0.99$$

We know that $z_{0.99} = 2.33$, ie $P(Z \le z_{0.99}) = 0.99$.
Thus

$$\frac{3-2.65}{0.85/\sqrt{n}} = 2.33$$
$$n = 32.02$$

so that $\boxed{n=32}$ will suffice.

Ice Cream Manufacturing:

$\mu = 12$
$\sigma = 1$
$n = 100$

a). $E(\bar{X}) = 12$

$SD(\bar{x}) = \frac{1}{10} = 0.1$

b). Since $n = 100 > 30$ we may approximate $\bar{x}$ to be normal. So

$P(11.8 \le \bar{X} \le 12.15) = P(-2 \le Z \le 1.5)$
$= \phi(1.5) - \phi(-2)$
$= 0.9332 - 0.0228$
$= \boxed{0.9104}$

c). We are using the Central Limit Theorem. The only assumption required is that $n > 30$.

Section 7.1

2. $(114.4, 115.6)$ $(114.1, 115.9)$

a). The sample mean is in the center of the interval so

$\bar{x} = \frac{114.4 + 115.6}{2} = \boxed{115}$

b). The first interval $(114.4, 115.6)$ has the 90% confidence. A higher confidence produces a wider interval.

Helium porosity:

Normal distribution with $\sigma = 0.69$.

a). $n = 25$, $\bar{x} = 4.56$

$$\left(4.56 \pm \frac{1.96(0.69)}{\sqrt{25}}\right) = (4.56 \pm 0.27) = \boxed{(4.29, 4.83)}$$

b). $n = 100$, $\bar{x} = 4.56$

$$\left(4.56 \pm \frac{1.96(0.69)}{\sqrt{100}}\right) = (4.56 \pm 0.135) = \boxed{(4.425, 4.695)}$$

c). When $n=25$, the width is 0.54. When $n=100$ the width is 0.27. Increasing the sample size decreases the interval width.

d). $n = 100$, $\bar{x} = 4.56$

$\alpha = 0.02 \Rightarrow z^*_{\frac{\alpha}{2}} = z^*_{0.01} = z_{0.99} = 2.33$ so that the interval is

$$\left(4.56 \pm \frac{(2.33)(0.69)}{\sqrt{100}}\right) = (4.56 \pm 0.16) = \boxed{(4.40, 4.72)}$$

e). With confidence level 95%, the width is 0.27. With confidence level 98%, the width is 0.32. Increasing the confidence increases the interval width.

f). $n = \left[\frac{2(1.96)(0.69)}{0.40}\right]^2 = 45.72$ so $\boxed{n=46}$ would suffice.

$\boxed{\text{Section 7.2}}$

24. $n = 56$

$\bar{x} = 8.17$

$s = 1.42$

$z_{\frac{\alpha}{2}} = 1.96$

Then the interval is

$$8.17 \pm 1.96\left(\frac{1.42}{\sqrt{56}}\right) = \boxed{(7.798, 8.542)}$$

We assume that the sample average elongation $\bar{x}$ is approximately normal.

Night School Students:

$n = 60$

$\bar{x} = 25.3$

$s = 4$

$z_{\frac{\alpha}{2}} = 2.58$

Then the interval is

$$25.3 \pm 2.58\left(\frac{4}{\sqrt{60}}\right) = \boxed{(23.97, 26.63)}$$