Section 7.3

28. a. $t_{0.1,15} = 1.341$

b. $t_{0.05,15} = 1.753$

c. $t_{0.05,25} = 1.708$

35. $n=15$, $\bar{x}=25$, $\sigma_{\bar{x}} = 3.5$ → use t!

a. We will find 95% CI. Then

$$\alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025 \Rightarrow t_{\frac{\alpha}{2},n-1} = 2.145$$

$$\therefore \bar{x} \pm t_{\frac{\alpha}{2},n-1}\frac{\sigma}{\sqrt{n}} = 25.0 \pm 2.145 \cdot \frac{3.5}{\sqrt{15}} = \boxed{(23.1,\ 26.9)}$$

b. For a 95% CI,

$$\bar{x} \pm t_{\frac{\alpha}{2},n-1}\, s\sqrt{1+\frac{1}{n}} = 25.0 \pm 2.145\,(3.5)\sqrt{1+\frac{1}{15}}$$
$$= \boxed{(17.2,\ 32.8)}$$

The PI is about 4x as wide as the CI.

37. $n=20$, $\bar{x}=0.9255$, $\sigma_{\bar{x}} = 0.0809$ ⇒ use t!

a. 95% CI: $\bar{x} \pm t_{\frac{\alpha}{2},n-1}\frac{\sigma}{\sqrt{n}} = \bar{x} \pm 2.093\,\frac{0.0809}{\sqrt{20}} = \boxed{(0.8876,\ 0.9634)}$

∴ there is a 95% likelihood that the mean cadence of mens' steps per second is in this interval.

b. 95% PI: $\bar{x} \pm t_{\frac{\alpha}{2}, n-1}\, \sigma \sqrt{1+\frac{1}{n}} = 0.9255 \pm 2.093(0.0809)\sqrt{1+\frac{1}{15}}$
$= \boxed{(0.7520, 1.0990)}$

$\therefore$ there is a 95% likelihood that the next randomly selected man has a cadence in this interval.

Supplemental Exercises

50. $n=5$, 95% CI $= (229.764, 233.504)$, Thus $\bar{x} = 231.634$.
To solve for s,

$$W = 2\, t_{0.025,4} \frac{s}{\sqrt{n}}$$
$$3.74 = 2(2.776)\frac{s}{\sqrt{5}}$$
$$s = 1.5063.$$

Thus for a 99% CI:

$$\alpha = 0.01 \Rightarrow \frac{\alpha}{2} = 0.005 \Rightarrow t_{\frac{\alpha}{2}, n-1} = 4.604$$

$$\therefore \bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}} = 231.634 \pm 4.604 \frac{1.5063}{\sqrt{5}} = \boxed{(228.53, 234.74)}$$

52. $n=5$, $\bar{x} = 24.3$, $\sigma_{\bar{x}} = 4.1$

a. 95% CI: $\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{\sigma}{\sqrt{n}} = 24.3 \pm 2.776 \frac{4.1}{\sqrt{5}}$
$= \boxed{(19.21, 29.39)}$

c. 95% PI: $\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{\sigma}{\sqrt{n}} = 24.3 \pm 2.776(4.1)\sqrt{1+\frac{1}{5}}$
$= \boxed{(11.83, 36.77)}$

Section 8.1

4. We generally formulate problems such that a Type I error is most serious. In the case that $H_a: \mu < 5$ a type I error is deciding the water is safe when it isn't. This is clearly the more serious of the two possible errors. Thus we will test

$$\boxed{H_a: \mu < 5}$$

6. $\begin{cases} H_0: & \mu = 40 \\ H_a: & \mu \neq 40 \end{cases}$

The alternative hypothesis states that a departure from 40 amps in either direction is unsafe.

Type I Error: We claim that $\mu < 40$ or $\mu > 40$ when in reality $\mu = 40$.
Type II Error: We fail to detect that μ has deviated from 40.

Section 8.2

15. a. $z_\alpha = 1.88 \Rightarrow \boxed{\alpha = 0.03}$

b. $z_\alpha = 2.75 \Rightarrow \boxed{\alpha = 0.003}$

c. $z_{\frac{\alpha}{2}} = 2.88 \Rightarrow \frac{\alpha}{2} = \boxed{0.004}$

20. $n = 50$, $\bar{x} = 738.44$, $\sigma_{\bar{x}} = 38.20$

$\underline{\alpha = 0.05}$

$$\begin{cases} H_0 : \mu = 750 \\ H_a : \mu < 750 \end{cases}$$

Test Stat: $Z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = -2.14$

RR: $Z \leq -Z_\alpha$

$Z \leq -1.645$

Since $-2.14 \leq -1.645$ we $\boxed{\text{reject the null}}$ and do not continue with the purchase.

$\underline{\alpha = 0.01}$

$$\begin{cases} H_0 : \mu = 750 \\ H_a : \mu < 750 \end{cases}$$

Test Stat: $Z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = -2.14$

RR: $Z \leq -Z_\alpha$

$Z \leq -2.33$

Since $-2.14 \nleq -2.33$ we $\boxed{\text{fail to reject the null}}$ and continue with the purchase.

29. $n=12$, $\bar{x}=249.7$, $\sigma_{\bar{x}}=145.1$, $\alpha=0.05$

a. $\begin{cases} H_0: \mu = 200 \\ H_a: \mu > 200 \end{cases}$

Jest Stat: $t = \dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = 1.17$

RR: $t \geqslant t_{\alpha, n-1}$
$t \geqslant 1.796$

Since $1.17 \not\geqslant 1.796$ we $\boxed{\text{fail to reject the null}}$ so that there is not significant evidence that the mean repair time is greater than 200 min.