

Stat 3115C - Exam 1

Name: KEY

Thursday, February 19, 2014
Time: 75 minutes
Instructor: Brittany Cuchta

Instructions:

- Do not open the exam until I say you may.
- Circle or box your final answer where appropriate.
- All work must clearly and legibly support your answer. Failure to show work sufficient to support your answer will result in the loss of points, even with the correct answer.
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

Materials Allowed:

- One calculator than cannot communicate with other devices. You may not share calculators during the exam.

Page:	1	2	3	4	5	6	7	Total
Points:	12	17	13	15	14	16	13	100
Score:								

1. (6 points) Circle the correct answer. Each part is worth two points.
- (a) True or **False**: A sample of 1000 college students is selected from all students registered at a certain college, and it turns out that 330 of them participate in intramural sports. We can conclude that the proportion of students at this college who participate in intramural sports is *exactly* 0.33.
- (b) **True** or False : The unit for the standard deviation is the same as the unit for the data values.
- (c) True or **False** : For any group of numbers, half of them will be below the mean.
2. (6 points) Circle the correct answer. Each part is worth two points.
- (a) When desired information is available for all objects in the population, we have a:
- A. population
 - B. sample
 - C**. census
 - D. boxplot
 - E. sample space
- (b) Which of the following statements regarding histograms are correct?
- A. A unimodal histogram is one that rises to a single peak and then declines, whereas a bimodal histogram is one that has two different peaks.
 - B. A unimodal histogram is skewed if the right or upper tail is stretched out compared to the left or lower tail.
 - C. A histogram is symmetric if the left half is a mirror image of the right half.
 - D**. All of the above.
 - E. Only (A) and (C), not (B).
- (c) Which of the following can be inferred from a boxplot?
- A**. Median
 - B. Mean
 - C. Variance
 - D. All of the above.
 - E. None of the above.

3. A study was conducted on S&T's campus to determine the extra walking distance (feet) that employees and students need to get around the various construction sites. Researchers collected the following information from a sample of 20 employees and students: extra walking distance (feet), building where employee or student spends most of their time (Rolla Building, Centennial Hall, or Butler Carlton Hall), and the level of inconvenience (not inconvenienced, mildly inconvenienced, or very inconvenienced).

- (a) (3 points) State the variables measured and whether they are categorical (C) or quantitative (Q).

Building (C) distance (Q)
inconvenience (C)

- (b) The following table gives the extra walking distance (feet) for the 20 employees/students.

41.2	45.8	50.2	57.1	57.9	58.8	58.9	62.1	63.4	64.6
65.2	65.6	67.4	68.6	69.1	71.3	73.2	74.7	79.4	88.1

Calculate the following by hand. You *must* show work. Failure to do so will result in zero points.

- (a) (2 points) Mean

$$\bar{x} = \frac{\sum_{i=1}^{20} x_i}{20} = \frac{1282.6}{20} = \boxed{64.13}$$

- (b) (2 points) Median

$$\frac{64.6 + 65.2}{2} = \frac{129.8}{2} = \boxed{64.9}$$

- (c) (3 points) Lower quartile (Q1)

$$\frac{57.9 + 58.8}{2} = \frac{116.7}{2} = \boxed{58.35}$$

- (d) (3 points) Upper quartile (Q3)

$$\frac{69.1 + 71.3}{2} = \frac{140.4}{2} = \boxed{70.2}$$

- (e) (4 points) Are there any outliers?

$$IQR = Q3 - Q1 = 70.2 - 58.35 = 11.85$$

$$\text{lower outliers: } < Q1 - 1.5 IQR = 58.35 - 17.775 = 40.58$$

$$\text{upper outliers: } > Q3 + 1.5 IQR = 70.2 + 17.775 = 87.975$$

$$\boxed{\therefore 88.1 \text{ is an outlier}}$$

4. A study was conducted at the Giddy Goat to investigate consumer buying habits. The following table gives data on the type of coffee that was purchased for 100 customers.

	Small	Medium	Large	
Regular	15	20	25	60
Decaf	26	12	2	40

- (a) (3 points) Calculate the probability that a randomly selected customer orders a small coffee.

$$P(\text{small}) = \frac{41}{100} = \boxed{0.41}$$

- (b) (3 points) Calculate the probability that a randomly selected customer orders a small decaf coffee.

$$P(\text{small and decaf}) = \frac{26}{100} = \boxed{0.26}$$

- (c) (3 points) Given that a customer orders a small coffee, what is the probability that the customer orders a decaf?

$$P(\text{decaf} | \text{small}) = \frac{P(\text{small and decaf})}{P(\text{small})}$$

$$= \frac{0.26}{0.41} = \boxed{0.63}$$

- (d) (4 points) Are the events ordering a decaf and ordering a small coffee independent? Show work to justify your answer.

$$P(\text{decaf} | \text{small}) = 0.63 \neq 0.4 = P(\text{decaf})$$

OR

$$P(\text{small and decaf}) = 0.26 \neq 0.164 = P(\text{small})P(\text{decaf})$$

\therefore not independent

5. A box contains 12 fuses. Seven of them are rated at 15 amperes (A) and the other five are rated at 20A. Three fuses are chosen at random.

(a) (4 points) What is the probability of getting a fuse of each type in the sample?

$$\begin{aligned}
 P(\text{one each type}) &= P(\text{one 15A and two 20A or two 15A and one 20A}) \\
 &= P(\text{one 15A and 2 20A}) + P(\text{two 15A and one 20A}) \\
 &= \frac{\binom{7}{1}\binom{5}{2} + \binom{7}{2}\binom{5}{1}}{\binom{12}{3}} = \frac{175}{220} = \boxed{0.7954}
 \end{aligned}$$

(b) (4 points) What is the probability of getting at least one 15A fuse?

$$\begin{aligned}
 P(\text{at least one 15A}) &= 1 - P(\text{no 15A}) \\
 &= 1 - \frac{\binom{7}{0}\binom{5}{3}}{\binom{12}{3}} \\
 &= 1 - 0.045 = \boxed{0.954}
 \end{aligned}$$

(c) (4 points) What is the probability of getting three fuses of the same type?

$$\begin{aligned}
 P(\text{all 15A or all 20A}) &= P(\text{all 15A}) + P(\text{all 20A}) \\
 &= \frac{\binom{7}{3}\binom{5}{0} + \binom{7}{0}\binom{5}{3}}{\binom{12}{3}} \\
 &= \frac{45}{220} = \boxed{0.2045}
 \end{aligned}$$

(d) (3 points) If we were to number the seven 15A fuses so that we can distinguish them, how many ways would there be to order these seven fuses?

$$\boxed{7! = 5040}$$

6. Courier Delivery Service in Rolla offers both express and standard delivery. 85% of parcels are sent by standard delivery and 15% are sent by express. Of those sent standard, 55% arrive the next day and of those sent express, 97% arrive the next day. A record of parcel delivery is chosen at random from the company's files.

- (a) (4 points) Calculate the probability that the parcel was shipped express and arrived the next day.

$$P(E \cap ND) = P(ND|E)P(E)$$

$$= 0.97(0.15)$$

$$= \boxed{0.1455}$$

- (b) (5 points) Calculate the probability that the parcel arrived the next day.

$$P(ND) = P(E \cap ND) + P(E' \cap ND)$$

$$= P(ND|E)P(E) + P(ND|E')P(E')$$

$$= 0.1455 + 0.55(0.85)$$

$$= \boxed{0.613}$$

- (c) (5 points) Given that the package arrived the next day, calculate the probability that it was sent express.

$$P(E|ND) = \frac{P(E \cap ND)}{P(ND)} = \frac{0.1455}{0.613} = \boxed{0.237}$$

*Note that you can use the tree diagram for this!

7. Delta Airlines has 10:00am flights from St. Louis to Atlanta, Boston, and Chicago. Let A denote the event that the Atlanta flight is full, and define the events B and C analogously for Boston and Chicago. Suppose $P(A) = 0.65$, $P(B) = 0.75$, $P(C) = 0.1$ and that the three events are mutually independent.

(a) (4 points) Calculate the probability that none of the flights are full.

$$\begin{aligned} P(\text{none full}) &= P(A' \cap B' \cap C') = P(A')P(B')P(C') \\ &= (1 - P(A))(1 - P(B))(1 - P(C)) \\ &= 0.35(0.25)(0.9) \\ &= \boxed{0.07875} \end{aligned}$$

(b) (4 points) Calculate the probability that at least one of the flights is full.

$$\begin{aligned} P(\text{at least one}) &= 1 - P(\text{none}) \\ &= 1 - 0.07875 \\ &= \boxed{0.92125} \end{aligned}$$

(c) (4 points) Calculate the probability that only the Atlanta flight is full.

$$\begin{aligned} P(A \cap B' \cap C') &= P(A)P(B')P(C') \\ &= 0.65(0.25)(0.9) \\ &= \boxed{0.14625} \end{aligned}$$

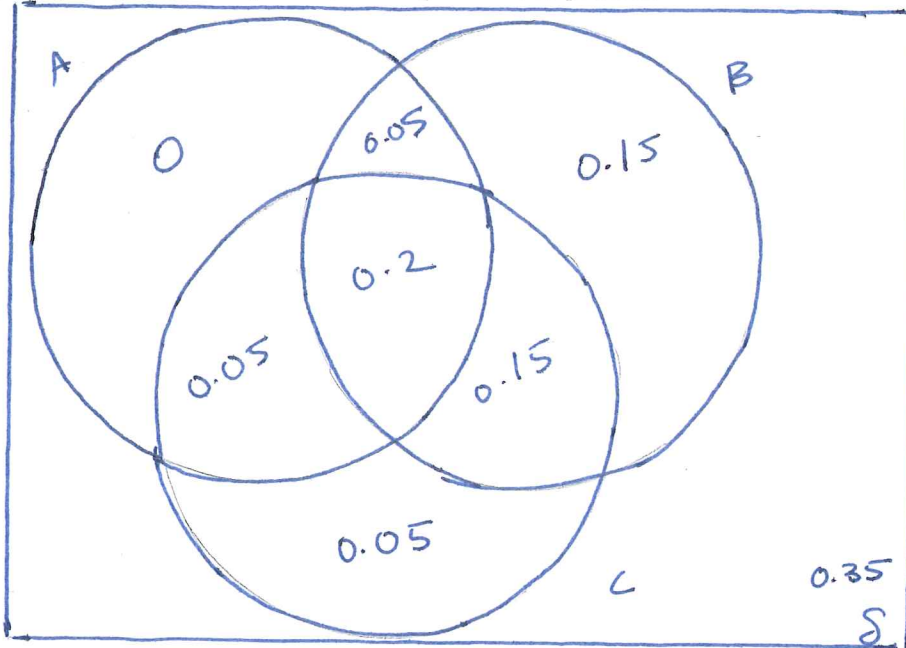
(d) (4 points) Are the events A and B mutually exclusive (disjoint)? Explain your answer.

$P(A \cap B) \neq 0$ therefore they are not disjoint

8. The three most popular options on apartments in Rolla are a garage (A), a dishwasher (B), and a laundry machine (C). You are given the following information about the preferences of the renters.

- 30% request A, 55% request B, and 45% request C.
- 25% request both A and B, 25% request both A and C, and 35% request both B and C.
- 20% request all three.

(a) (7 points) Fill out the completed Venn Diagram for this problem.



(b) (3 points) Calculate the probability that the next renter will request exactly one of these three options.

$$P(\text{exactly one}) = 0 + 0.05 + 0.15$$

$$= \boxed{0.2}$$

(c) (3 points) Given that the renter requests a laundry machine (C), what is the probability that the renter will request a garage (A)?

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{0.25}{0.45} = \boxed{0.55}$$