Stat 3115C - Exam 2

Name: KEY

Tuesday, April 7, 2014
Time: 75 minutes
Instructor: Brittany Cuchta

Instructions:

- Do not open the exam until I say you may.
- Circle or box your final answer where appropriate.
- All work must clearly and legibly support your answer. Failure to show work sufficient to support your answer will result in the loss of points, even with the correct answer.
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

Materials Allowed:

- One calculator than cannot communicate with other devices. You may not share calculators during the exam.

Page:	1	2	3	4	5	6	7	Total
Points:	12	16	13	12	12	18	17	100
Score:								

1. (4 points) Circle the correct answer. Each question is worth two points.

   (a) True or False : A discrete random variable can assume only a finite number of possible values.

   (b) True or False: X and Y are independent if there exist some x, y such that $p(x, y) = p_X(x)p_Y(y)$.

2. (4 points) Fill in the appropriate answer in the provided blank. Each question is worth two points.

   (a) Let X be a discrete random variable with $V(X) = 5$. Then $V(2X + 20.2)$ is 20.

   $V(aX+b) = a^2 V(X)$

   (b) If X is a continuous random variable and c is a real number, then $P(X = c)$ is 0.

3. (4 points) Circle the best answer. Each question is worth two points.

   (a) Which of the following is true for a continuous random variable X with pdf $f(x)$?

   A. For any two real numbers a and b such that $a \le b$, $P(a \le X \le b) = \int_a^b f(x)\, dx$.

   B. $f(x) \ge 0$.

   C. $\int_{-\infty}^{\infty} f(x)\, dx = 1$.

   D. All of the above.

   E. None of the above.

   F. Only A and B are true, not C.

   (b) Let X be a continuous random variable with probability density function (pdf) $f(x)$ and cumulative density function (cdf) $F(x)$. Then for any two numbers a and b with a strictly less than b, which of the following is true?

   A. $P(a \le X \le b) = F(a) - F(b)$.

   B. $P(X > a) = 1 - F(a)$.

   C. $P(X > b) = F(b) - 1$.

   D. All of the above.

   E. None of the above.

   F. Only A and B are true, not C.

4. Suppose X is a discrete random variable whose cdf is given below.

$$F(x) = \begin{cases} 0 & \text{if } x < -2 \\ 0.2 & \text{if } -2 \leq x < 2 \\ 0.5 & \text{if } 2 \leq x < 3 \\ 0.9 & \text{if } 3 \leq x < 6 \\ 1 & \text{if } x \geq 6 \end{cases}$$

(a) (4 points) Find the probability that X is greater than 3.

$$P(X > 3) = 1 - P(X \leq 3)$$

$$= 1 - F(3)$$

$$= 1 - 0.9$$

$$= \boxed{0.1}$$

(b) (6 points) Give the pmf of X.

x	-2	2	3	6
p(x)	0.2	0.3	0.4	0.1

5. (6 points) Walt owns a car wash and he has noticed that the customers' arrival has a **normal distribution** with a mean time of 4.5 hours after opening and a standard deviation of 1 hour. If the car wash opens at 8:00 am, find the probability that a customer will arrive between 11am and 2pm.

11am – 3hrs after opening
1pm – 6 hrs after opening

$$P(3 \leq X \leq 6) = P(-1.5 \leq Z \leq 1.5) = \phi(1.5) - \phi(-1.5)$$

$$= 0.9332 - 0.0668$$ Table A3

$$= \boxed{0.8664}$$

6. Assume that 1 out of every 20 people will have an adverse reaction to a new drug. The drug company takes a random sample of 65 patients who are good candidates and gives them the drug. Let Y be the number of patients that have an adverse reaction to the drug among the 65 sampled.

(a) (3 points) What kind of distribution does Y have? Be sure to give both the name and parameter values.

$n = 65, \ p = \frac{1}{20} = 0.05$

$\underline{X} \sim \text{Binomial}(65, 0.05)$

(b) (5 points) If the company has to pay \$15,000 to every patient in the sample who suffers an adverse effect, how much money should the company expect to pay?

$E(15000\underline{X}) = 15000\,E(\underline{X}) = 15000 \cdot n \cdot p$

$= \boxed{\$48,750}$

(c) (5 points) Suppose the company had just wanted us to give a rough approximation of the amount of money they should expect to pay. What approximated distribution could you use? Is the approximation appropriate given our assumptions? Be sure to list the distribution name, parameter values, and assumptions that must be met.

If $n > 50$, $np < 5$ then we may approximate with poisson distribution.

$n = 65 > 50$ ✓

$np = 3.25 < 5$ ✓

∴ we meet the conditions and may approximate with poisson distribution.

7. Jesse is a salesman and from past experience, he knows that 32% of people he peddles to will buy his new product. **For *each part* of this problem, define the random variable needed for the calculation, giving both the distribution and parameter values.**

   (a) (6 points) What is the probability that the fifth person to buy will be the eighth person Jesse has talked to?

   $r = 5$, $p = 0.32$

   $X \sim NB(5, 0.32)$

   Talk to 8 people with 5 successes ⇒ 3 failures

   $$P(X=3) = \binom{3+5-1}{5-1} 0.32^5 \, 0.68^3$$

   $$= \binom{7}{4} 0.32^5 \cdot 0.68^3$$

   $$= \boxed{0.0369}$$

   (b) (6 points) Jesse gets \$170 from each successful sale. His boss has told him that his goal for the day is \$5100 in sales, and once he has sold enough products, he will be able to go home. How many people should Jesse expect to talk to before he goes home?

   $$\frac{5100}{170} = 30 \text{ sales needed}$$

   $Y \sim NB(30, 0.32)$.

   $$E(Y) = \frac{r(1-p)}{p} = \frac{30(0.68)}{0.32} = 63.75 = \text{\# failures}$$

   total number of people $= S + F = 30 + 63.75 = 93.75$

   $\boxed{\therefore \text{ Jesse should expect to talk to 93 or 94 people}}$

8. Hank the Geologist has collected 25 minerals, 12 of which are Hank's favorite, the mineral hanksite. He has asked his lab assistant Marie to take a sample of 7 for analysis. Let X be the number of hanksite specimens chosen for analysis.

   (a) (3 points) What kind of distribution does X have? Be sure to list both the distribution and parameter values.

   $N=25,\ M=12,\ n=7$

   $\underline{X} \sim \text{Hyper}(25, 12, 7)$

   (b) (4 points) What is the expected number of hanksite specimens among the seven that Marie draws?

   $$E(X) = n \cdot \frac{M}{N} = \frac{7 \cdot 12}{25} = \boxed{3.36}$$

   (c) (5 points) Find the probability all of the selected minerals are hanksite.

   $$P(X=7) = \frac{\binom{12}{7}\binom{13}{0}}{\binom{25}{7}} = \frac{792}{480700} = \boxed{0.0016}$$

9. Flynn is at a breakfast bar, where bacon and eggs are the two most popular dishes. Let X be the number of bacon pieces ordered by a customer and Y be the number of eggs ordered. The joint pmf of X and Y is given in the table below:

	$p(x,y)$	y = 0	1	2	3
x	0	0.01	0.02	0.05	0.09
	2	0.1	0.15	0.15	0.05
	4	0.03	0.02	0.04	0.01
	5	0.1	0.1	0.07	0.01

(a) (5 points) Give the marginal pmf of X.

X	0	2	4	5
$p_X(x)$	0.17	0.45	0.1	0.28

(b) (5 points) Find the probability that a customer will take at least 2 pieces of bacon and at least 1 egg.

$$P(X \geq 2, Y \geq 1) = p(2,1) + p(4,1) + p(5,1) + p(2,2) + \dots$$

$$= \boxed{0.6}$$

(c) (4 points) If bacon costs \$0.50 per piece, find the expected customer cost for bacon.

$$E(0.5X) = 0.5E(X) = 0.5 \sum_x x\, p_X(x)$$

$$= 0.5(2.7) = \boxed{\$1.35}$$

(d) (4 points) Are X and Y independent? Support your answer.

X and Y are not independent. Note that

$p(0,0) = 0.01 \neq 0.0255 = p_X(0)\, p_Y(0)$.

10. The reaction time (in seconds) to a certain stimulus is a continous random variable with the pdf below:

$$f(x) = \begin{cases} \frac{k}{x^2} & \text{if } 1 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) (3 points) Show that the value of $k = \frac{4}{3}$ that makes $f(x)$ a legitimate pdf. *You must show work!*

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_1^4 \frac{4}{3} x^{-2}\,dx = \frac{4}{3}\left[-x^{-1}\right]_{x=1}^{4} = \frac{4}{3}\left(-\frac{1}{4} + \frac{4}{4}\right)$$

$$= \frac{4}{3} \cdot \frac{3}{4}$$

$$= 1 \checkmark \quad \therefore f(x) \text{ is a valid pdf}$$

(b) (5 points) Obtain the cdf and write in functional form.

$$F(x) = \int_1^x \frac{4}{3} y^{-2}\,dy = -\frac{4}{3} y^{-1}\Big|_{x=1}^{x} = -\frac{4}{3}x^{-1} + \frac{4}{3}$$

$$\therefore F(x) = \begin{cases} 0 & , \quad x < 1 \\ \frac{4}{3}(1 - x^{-1}) & , \quad 1 \le x \le 4 \\ 1 & , \quad x > 4 \end{cases}$$

(c) (5 points) Using either the cdf or pdf, what is the probability that the reaction time is between 1.5 and 2.5 seconds?

$$P(1.5 \le X \le 2.5) = P(X \le 2.5) - P(X \le 1.5)$$

$$= F(2.5) - F(1.5)$$

$$= 0.8 - 0.\overline{4} = \boxed{0.3\overline{5}}$$

(d) (4 points) Find the average reaction time.

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_1^4 \frac{4}{3} x^{-1}\,dx = \frac{4}{3} \ln x \Big|_1^4 = \frac{4}{3}(\ln 4) = \boxed{1.848}$$