Stat 3115D - Exam 2

Name: KEY

Wednesday, April 8, 2014
Time: 50 minutes
Instructor: Brittany Cuchta

Instructions:

- Do not open the exam until I say you may.
- Circle or box your final answer where appropriate.
- All work must clearly and legibly support your answer. Failure to show work sufficient to support your answer will result in the loss of points, even with the correct answer.
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

Materials Allowed:

- One calculator than cannot communicate with other devices. You may not share calculators during the exam.

Page:	1	2	3	4	5	6	Total
Points:	21	15	14	14	20	16	100
Score:							

1. (4 points) Circle the correct answer. Each question is worth two points.

   (a) True or False : A discrete random variable can assume only a finite number of possible values.

   (b) True or False : X and Y are independent if there exist *some* x, y such that $p(x, y) = p_X(x)p_Y(y)$.

2. (4 points) Fill in the appropriate answer in the provided blank. Each question is worth two points.

   (a) Let X be a discrete random variable with $V(X) = 5$. Then $V(2X + 20.2)$ is 20.

   $V(aX+b) = a^2V(X)$

   (b) If X is a continuous random variable and c is a real number, then $P(X = c)$ is 0.

3. Hank the Geologist has collected 25 minerals, 12 of which are Hank's favorite, the mineral hanksite. He has asked his lab assistant Marie to take a sample of 7 for analysis. Let X be the number of hanksite specimens chosen for analysis.

   (a) (3 points) What kind of distribution does X have? Be sure to list both the distribution and parameter values.

   $$X \sim \text{Hyper}(\overset{n}{7}, \overset{M}{12}, \overset{N}{25})$$

   (b) (5 points) What is the expected number of hanksite specimens among the seven that Marie draws?

   $$E(X) = n \cdot \frac{M}{N} = 7 \cdot \frac{12}{25} = \boxed{3.36}$$

   (c) (5 points) Find the probability all of the selected minerals are hanksite.

   $$P(X=7) = \frac{\binom{12}{7}\binom{13}{0}}{\binom{25}{7}} = \frac{792}{480{,}700} = \boxed{0.0016}$$

$$X \sim N(\underset{\mu}{61{,}500}; \underset{\sigma}{9.500})$$

4. Benny is a mechanic who helps install emissions control air pumps. If a pump fails before the vehicle in which it is installed has covered 53,000 miles, regulations in the state of New Mexico require that it be replaced at no cost to the vehicle owner. The number of miles a pump operates before becoming ineffective has been found to be normally distributed with an average of 61,500 miles and a standard deviation of 9,500 miles.

   (a) (5 points) What percentage of the pumps Benny installs will have to be replaced at no charge to the vehicle owner?

$$P(X < 53000) = P\left(Z < \frac{53000 - 61500}{9500}\right)$$

$$= P(Z < -0.89)$$

$$= \boxed{0.1867} \quad \text{Jable A3}$$

   (b) (5 points) What is the number of miles such that 69.5% of all pumps will last no longer than that value?

$$P(Z < \hat{z}) = 0.695$$

$$z = \frac{x - \mu}{\sigma} \qquad \hat{z} = 0.51 \quad \text{Jable A3}$$

$$z\sigma + \mu = x \qquad \hat{z}\sigma + \mu = 0.51(9500) + 61500$$

$$\boxed{\hat{x} = 66{,}345 \text{ mi}}$$

   (c) (5 points) What is the probability that the pump will last for no more than 100,000 miles?

$$P(X \leq 100{,}000) = P\left(Z \leq \frac{100000 - 61500}{9500}\right)$$

$$= P(Z \leq 4.05)$$

$$= \boxed{1.}$$

$X \sim \text{Poisson}(4)$

5. Francesca is a secretary at an office. Let X denote the number of people who call the office in a one minute period. In her experience, Francesca notes that X follows a Poisson distribution with a mean value of 4 calls per minute.

   (a) (4 points) Find the probability that no one calls the office within the next minute.

$$P(X=0) = \frac{e^{-\mu}\mu^{x}}{x!} = \frac{e^{-4}\cdot 4^{0}}{0!} = \boxed{0.0183}$$

   (b) (5 points) Find the probability that at least 1 person calls during the next minute.

$$P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0)$$

$$= 1 - 0.0183$$

$$= \boxed{0.9817}$$

   (c) (5 points) Find the probability that at most 2 people call in the next 30-second period.

$$\mu = \alpha t = 4 \cdot \frac{1}{2} = 2.$$

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \sum_{x=0}^{2} \frac{e^{\cdot\cdot}\,{}^{x}}{x!} =$$

$$= \boxed{0.6767}$$

6. Jesse is a salesman and from past experience, he knows that 32% of people he peddles to will buy his new product. **For *each part* of this problem, define the random variable needed for the calculation, giving both the distribution and parameter values.**

   (a) (7 points) What is the probability that the fifth person to buy will be the eighth person Jesse has talked to?

$X \sim NB(5, 0.32)$

five people buy & eight people talked to
$\Rightarrow$ 3 failures
5 success

$$P(X=3) = \binom{5+3-1}{5-1} 0.32^5 \cdot 0.68^3$$

$$= \binom{7}{4} 0.32^5 \cdot 0.68^3$$

$$= \boxed{0.0369}$$

   (b) (7 points) Jesse gets \$170 from each successful sale. His boss has told him that his goal for the day is \$5100 in sales, and once he has sold enough products, he will be able to go home. How many people should Jesse expect to talk to before he goes home?

$\frac{5100}{170} = 30$ sales needed

$Y \sim NB(30, 0.32)$

$$E(Y) = \frac{r(1-p)}{p} = \frac{30\ (0.68)}{0.32} = 63.75 \text{ failures}$$

Total people talked to = success + failures

$= 30 + 63.75$

$= 93.75$

$\boxed{\therefore \text{ Jesse will talk to about 93 people}}$

7. Flynn is at a breakfast bar, where bacon and eggs are the two most popular dishes. Let X be the number of bacon pieces ordered by a customer and Y be the number of eggs ordered. The joint pmf of X and Y is given in the table below:

	$p(x,y)$	y = 0	1	2	3
x	0	0.01	0.02	0.05	0.09
	2	0.1	0.15	0.15	0.05
	4	0.03	0.02	0.04	0.01
	5	0.1	0.1	0.07	0.01

(a) (5 points) Give the marginal pmf of Y.

y	0	1	2	3
$p_Y(y)$	0.24	0.29	0.31	0.16

(b) (5 points) Find the probability that a customer will take at least 1 piece of bacon and at least 2 eggs.

$P(X \geq 1, Y \geq 2) = p(2,2) + p(4,2) + p(5,2) + p(2,3) + \cdots$

$= \boxed{0.33}$

(c) (5 points) If eggs are \$1.25 each, find the expected cost of a customer's egg purchase.

$E(1.25Y) = 1.25\,E(Y) = 1.25\left(\sum_y y\,p_Y(y)\right)$

$= 1.25(1.39)$

$= \boxed{\$1.74}$

(d) (5 points) Are X and Y independent? Support your answer.

no, X and Y are not independent.

Note: $p(0,0) = 0.01 \neq 0.0408 = p_X(0)\,p_Y(0)$.

8. The reaction time (in seconds) to a certain stimilus is a continous random variable with the pdf below:

$$f(x) = \begin{cases} \frac{k}{x^2} & \text{if } 1 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) (5 points) Show that the value of $k = \frac{4}{3}$ that makes $f(x)$ a legitimate pdf. *You must show work!*

we require : $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_1^4 \frac{4}{3} \cdot x^{-2}\,dx = -\frac{4}{3} x^{-1} \Big|_{x=1}^{4} = -\frac{4}{3} \cdot \frac{1}{4} + \frac{4}{3} \cdot 1 = \frac{4}{3} - \frac{1}{3} = 1. ✓$$

$\therefore$ $f(x)$ with $k = \frac{4}{3}$ is a valid pdf.

(b) (6 points) Obtain the cdf and write in functional form.

$$F(x) = \int_{-\infty}^{x} f(y)\,dy = \int_1^x \frac{4}{3} y^{-2}\,dy = -\frac{4}{3} y^{-1} \Big|_{y=1}^{x} = -\frac{4}{3} x^{-1} + \frac{4}{3} \cdot 1$$

$$\boxed{\therefore\ F(x) = \begin{cases} 0 & , \ x < 1 \\ \frac{4}{3}(1 - x^{-1}) & , \ 1 \le x \le 4 \\ 1 & , \ x > 4 \end{cases}}$$

(c) (5 points) Find the average reaction time.

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_1^4 x \cdot \frac{4}{3} x^{-2}\,dx = \int_1^4 \frac{4}{3} x^{-1}\,dx$$

$$= \frac{4}{3} \ln x \Big|_{x=1}^{4} = \frac{4}{3} \ln 4 = \boxed{1.848}$$