Word Problem Formulæ for §6.7, §6.8

§6.7 — Financial Models

1. Simple Interest Formula:
   Let P be the principle amount borrowed or invested, r be the rate at which interest is charged (or accrued), and t be the time (in years) that passes before you wish to calculuate interest gained, denoted by I:

$$I = Prt$$

2. Compound Interest Formula:
   Let P be the principle amount borrowed or invested, r be the rate at which interest is charged (or accrued), n be the number of times per year that the interest is compounded, and t be the time (in years) that passes before you wish to see the total amount of your invested, denoted by A:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

3. Continuous Compounding Interest Formula:
   Let P be the principle amount borrowed or invested, r be the rate at which interest is charged (or accrued), and t be the time (in years) that passes before you wish to see the total amount of your investment, denoted by A:

$$A = Pe^{rt}$$

4. Effective Rate of Interest:
   Let r_e be the effective interest rate you wish to calculuate for a given compounding interest rate r.

   (a) Formula for Compounding n times a year:

$$r_e = \left(1 + \frac{r}{n}\right)^{n} - 1$$

   (b) Formula for Continuous Compounding:

$$r_e = e^{r} - 1$$

§6.8 — Growth and Decay Models

1. Uninhibited Growth or Decay Formula:
   Let $A(t)$ be the amount of substance in existence at time t. At time $t = 0$, the amount of substance is A_0. Then the model

   $$A(t) = A_0 e^{kt}$$

   represents either growth or decay. If $k > 0$ then the model is for exponential **growth**. If $k < 0$ then the model is for exponential **decay**.

2. Newton's Law of Cooling:
   Let there exist an object whose temperature u can be modeled as a function of time t. Then, if T is the constant temperature of the object's surroundings, u_0 is the temperature of the object at time $t = 0$, and $k < 0$ is a constant so that

   $$u(t) = T + (u_0 - T)\, e^{kt}$$

3. Inhibited Growth or Decay Formula (Logistic Formula):
   Let $P(t)$ be the amount of a substance after a certain amount of time t, let a and c be positive constants, and let b be a constant. Then

   $$P(t) = \frac{c}{1 - ae^{-bt}}.$$

   If $b > 0$ then this is a **decay** formula. If $b < 0$ this is a **growth** formula. Note that $P(t) \to c$ as $t \to \infty$. For growth models, c is called the *carrying capacity*.