Math 1160A — Exam 3

Name: KEY

Friday, July 22, 2016
Time: 60 minutes
Instructor: Brittany Cuchta

Instructions:

- Do not open the exam until I say you may.
- All cell phones and other electronic noisemaking devices must be turned off or completely silenced (i.e., not on vibrate) for the duration of the exam.
- The exam *must* be taken in pencil. Using a pen on the exam will result in the loss of points.
- Failure to follow directions specific to a problem will result in the loss of points.
- Circle or box your final answer where appropriate. Put your final answer in the provided space when available. Failure to do so will result in points being deducted.
- Show **all** work. Full credit will only be given if work is shown which **fully and clearly** justifies your answer. I reserve the right to not grade a problem which I cannot read.
- Answers must be exact (like $\sqrt{2}$), not approximate (like 1.414), unless a problem specifically indicates otherwise.
- All final answers must be simplified unless otherwise specified. **Rationalization is not required unless otherwise specified.**
- If you run out of room, use the back of the page and indicate this on the question.
- As always, you are expected to exhibit academic integrity during the exam.

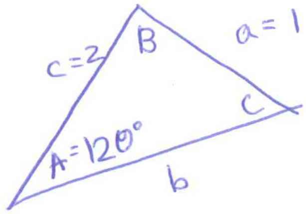
Materials Allowed:

- One calculator that cannot communicate with other devices. You may not share calculators during the exam.

Page:	1	2	3	4	5	Total
Points:	22	25	32	17	4	100
Score:						

1. (8 points) Solve the following triangle. There may be one, two, or no solutions. Be sure to give reasoning supporting your argument for the number of triangles. Round all answers to **two** decimals.

$a = 1,\ c = 2,\ A = 120°$



$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\sin C = \frac{c \sin A}{a} = 2\sin(120°) = \sqrt{3} > 1$$

Since $-1 \leq \sin(x) \leq 1$, there is no angle C st $\sin C = \sqrt{3}$. Thus there is no $\boxed{\text{solution}}$

Number of Triangles: 0 $A =$ / $C =$ / $a =$ /

$A' =$ ______ $C' =$ ______ $a' =$ ______

2. (14 points) Solve and find the area of the following triangle. Round all answers to **two** decimal places.

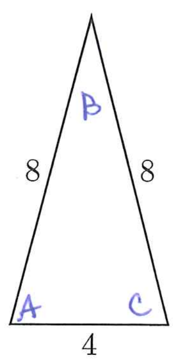
$c^2 = a^2 + b^2 - 2ab\cos C$

$8^2 = 8^2 + 4^2 - 2(8)(4)\cos C$

$\cos C = 0.25 \rightarrow C = 75.52°$

Since $a = c$, $A = C$ and $A = 75.52$.

Finally, $B = 180 - A - C = 28.96°$



Heron:

$s = \frac{1}{2}(a+b+c) = 10$

$K = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10(2)(2)(6)}$

$= 15.49$

OR $K = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(8)(4)\sin(75.52°)$

$= 15.49$

$A =$ 75.52° $B =$ 28.96° $C =$ 75.52° Area = 15.49

3. (6 points) Transform the following point from polar to rectangular coordinates. Give **exact** values.

$$\left(3, -\frac{\pi}{4}\right)$$
$r \quad \theta$

$x = r\cos\theta = 3\cos\left(-\frac{\pi}{4}\right) = \frac{3}{\sqrt{2}}$

$y = r\sin\theta = 3\sin\left(-\frac{\pi}{4}\right) = -\frac{3}{\sqrt{2}}$

Solution: $\left(\frac{3}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right)$

4. Transform the following points from rectangular to polar coordinates. If necessary, round your final answer to **one** decimal point.

(a) (8 points) $(-3, -3\sqrt{3})$
$x \quad y$

$r = \sqrt{x^2+y^2}$
$= \sqrt{(-3)^2 + (-3\sqrt{3})^2}$
$= \sqrt{36}$
$= 6$

$\sin\theta = \frac{y}{r} = -\frac{\sqrt{3}}{2}$
$\cos\theta = \frac{x}{r} = -\frac{1}{2}$ $\rightarrow \theta = \frac{4\pi}{3}$

OR $\theta = \tan^{-1}\left(\frac{y}{x}\right) + \pi = \tan^{-1}(\sqrt{3}) + \pi = \frac{4\pi}{3}$

because QIII

Solution: $\left(6, \frac{4\pi}{3}\right)$

(b) (8 points) $(8.3, 4.2)$
$x \quad y$

$r = \sqrt{x^2+y^2}$
$= \sqrt{(8.3)^2 + (4.2)^2}$
$= \sqrt{86.53}$
$= 9.30$

$\sin\theta = \frac{y}{r} = \frac{4.2}{9.30}$
$\cos\theta = \frac{x}{r} = \frac{8.3}{9.30}$ $\rightarrow \theta = 0.469$ (calculator)

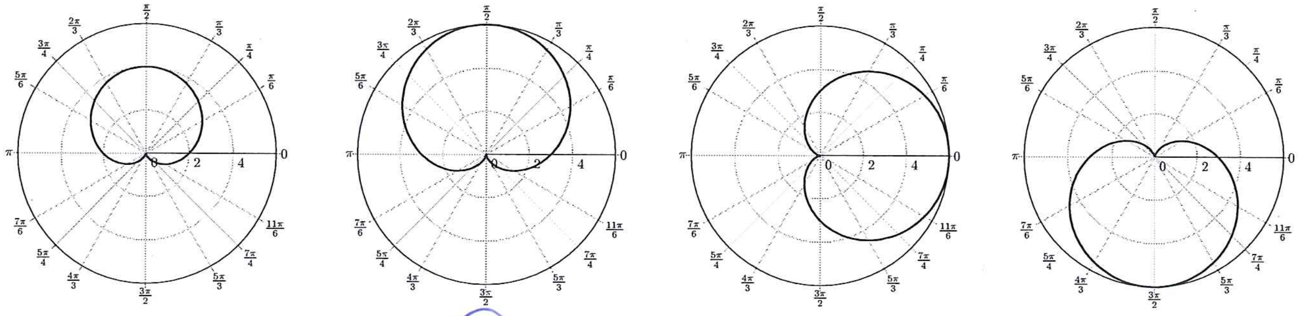
OR $\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4.2}{8.3}\right) = 0.469$

Solution: $(9.30, 0.47)$

5. (3 points) A polar equation is given below. Choose the graph which best describes the equation. Partial credit will not be given.

$3 + 3\sin\left(\frac{\pi}{2}\right) = 3 + 3(1) = 6 \rightarrow$ B!

$$r = 3 + 3\sin\theta$$



A B C D

6. (7 points) Transform the following equation from polar to rectangular. Then, **completely** describe the equation.

$$r = 5$$

$r=5$

$\sqrt{x^2+y^2}=5$

$x^2+y^2=25$

Equation: $x^2+y^2=25$ Shape: circle, center $(0,0)$, radius 5

7. (10 points) Write the following in polar form. Give the argument of z.

$$z = 4 - 4i$$

$r=\sqrt{x^2+y^2}=\sqrt{16+16}=\sqrt{32}$

$=4\sqrt{2}$

$\sin\theta=\frac{y}{r}=\frac{-4}{4\sqrt{2}}=-\frac{1}{\sqrt{2}}$ $\rightarrow \theta=\frac{7\pi}{4}$

$\cos\theta=\frac{x}{r}=\frac{4}{4\sqrt{2}}=\frac{1}{\sqrt{2}}$

or $\theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}(-1)=-\frac{\pi}{4} \rightsquigarrow \frac{7\pi}{4}$

Solution: $4\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right)$ Argument: $\frac{7\pi}{4}$

8. (15 points) Write the following expression in $a+bi$ form. Use De Moivre's formula. **No credit will be awarded for other methods.**

$$z^3 = (-1+i)^3$$

$r=\sqrt{1+1}=\sqrt{2}$

$\sin\theta=\frac{1}{\sqrt{2}}$ $\rightarrow \theta=\frac{3\pi}{4}$

$\cos\theta=-\frac{1}{\sqrt{2}}$

or $\theta=\tan^{-1}(-1)+\pi=\frac{3\pi}{4}$

because QII

$z=\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right)$

$z^3=(\sqrt{2})^3\left(\cos\left(3\cdot\frac{3\pi}{4}\right)+i\sin\left(3\cdot\frac{3\pi}{4}\right)\right)$

$=2\sqrt{2}\left(\cos\left(\frac{9\pi}{4}\right)+i\sin\left(\frac{9\pi}{4}\right)\right)$

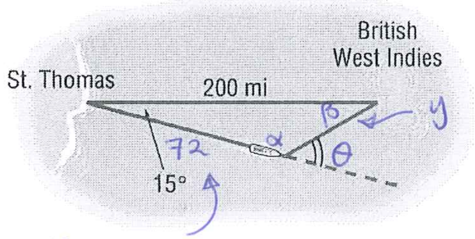
$=2\sqrt{2}\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)$

$=2+2i$

Solution: $2+2i$

9. (17 points) A sailboat leaves St. Thomas bound for an island in the British West Indies, 200 miles away. Maintaining a constant speed of 18 miles per hour, but encountering heavy crosswinds and strong currents, the crew finds that after 4 hours the sailboat is off course by 15°. Through what angle should the sailboat turn to correct its course? Round your answer to **two** decimal places.

$\theta = 180 - \alpha$



$18(4) = 72$

$y^2 = 72^2 + 200^2 - 2(72)(200)\cos(15°)$

$= 17365.3362$

$y = 131.7776$

$\boxed{\text{Law of Sines}}$

$\frac{\sin 15°}{y} = \frac{\sin\alpha}{200}$

$\sin\alpha = \frac{200\sin 15°}{y} = 0.39281$

$\alpha = 23.1296°$

☆ Is this the reference angle for α?

If $\alpha = 23.13°$, then $\beta = 141.87°$ and $\beta > \alpha$. But $200 > 72$ so we require $\alpha > \beta$. Thus $23.13°$ is the reference angle and

$\alpha = 156.87°$

so $\theta = 180 - \alpha = 23.13°$

$\boxed{\text{Law of Cosines}}$

$200^2 = 72^2 + y^2 - 2(72)(y)\cos\alpha$

$\cos\alpha = \frac{72^2 + y^2 - 200^2}{2(72)(y)} = -0.9196$

$\alpha = 156.87°$

$\therefore\ \theta = 180 - \alpha = 23.13°$

Solution: 23.13°