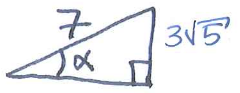
Math 1160 — Summer 2016 Name: KEY

In-Class Worksheet — Chapter 8

Complete the following worksheet for up to 5 bonus points. Write your answer in the provided spaces.

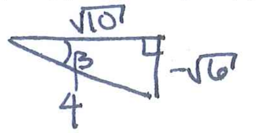
1. If $\cos\alpha = \frac{2}{7}$, $0 < \alpha < \frac{\pi}{2}$, and $\cos\beta = \frac{\sqrt{10}}{4}$, $\frac{3\pi}{2} < \beta < 2\pi$, find the following:

   A. $\sin(\alpha+\beta)$ B. $\sin(\alpha-\beta)$ C. $\cos(\alpha+\beta)$ D. $\sin(2\beta)$ E. $\cos(2\alpha)$ F. $\sin\left(\frac{\alpha}{2}\right)$ G. $\cos\left(\frac{\beta}{2}\right)$




$\alpha \in QI$

$\frac{\alpha}{2} \in QI$




$\beta \in Q\,IV$

$\beta \in QII$

D. $\sin(2\beta) = 2\sin\beta\cos\beta = 2\left(-\frac{\sqrt{6}}{4}\right)\left(\frac{\sqrt{10}}{4}\right)$

$= -\frac{2\sqrt{60}}{16} = \frac{-4\sqrt{15}}{16} = \boxed{-\frac{\sqrt{15}}{4}}$ D

A. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$= \frac{3\sqrt{5}}{7}\cdot\frac{\sqrt{10}}{4} + \frac{-\sqrt{6}}{4}\cdot\frac{2}{7}$

$= \frac{3\sqrt{50} - 2\sqrt{6}}{28}$

$= \boxed{\frac{15\sqrt{2} - 2\sqrt{6}}{28}}$ A.

E. $\cos(2\alpha) = 2\cos^2\alpha - 1$

$= 2\left(\frac{2}{7}\right)^2 - 1$

$= 2\left(\frac{4}{49}\right) - 1$

$= \frac{8}{49} - \frac{49}{49}$

$= \boxed{-\frac{41}{49}}$ E.

B. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$= \boxed{\frac{15\sqrt{2} + 2\sqrt{6}}{28}}$ B.

F. $\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\frac{2}{7}}{2}}$

$= \boxed{\sqrt{\frac{5}{14}}}$ F

C. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

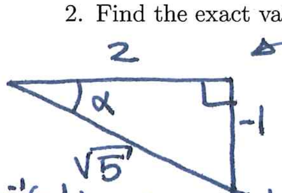
$= \frac{2}{7}\cdot\frac{\sqrt{10}}{4} - \frac{3\sqrt{5}}{7}\cdot\frac{-\sqrt{6}}{4}$

$= \boxed{\frac{2\sqrt{10} - 3\sqrt{30}}{28}}$ C.

G. $\cos\left(\frac{\beta}{2}\right) = \pm\sqrt{\frac{1+\cos\beta}{2}} = -\sqrt{\frac{1+\frac{\sqrt{10}}{4}}{2}}$

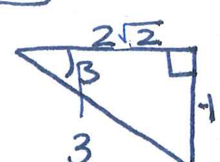
$= -\sqrt{\frac{4+\sqrt{10}}{8}} = \boxed{-\frac{\sqrt{4+\sqrt{10}}}{2\sqrt{2}}}$ G.

2. Find the exact value of $\cos\left[\tan^{-1}\left(-\frac{1}{2}\right) - \csc^{-1}(-3)\right]$.




$\alpha = \tan^{-1}\left(-\frac{1}{2}\right) \rightarrow \tan\alpha = -\frac{1}{2}$

$\alpha \in Q\,IV$




$\beta = \csc^{-1}(-3) \rightarrow \csc\beta = -3$

$\rightarrow \sin\beta = -\frac{1}{3}$

$\beta \in Q\,IV$

$\cos\left(\tan^{-1}\left(-\frac{1}{2}\right) - \csc^{-1}(-3)\right) = \cos(\alpha - \beta)$

$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \frac{2}{\sqrt{5}}\cdot\frac{2\sqrt{2}}{3} + \frac{-1}{\sqrt{5}}\cdot\frac{-1}{3}$

$= \boxed{\frac{4\sqrt{2}+1}{3\sqrt{5}}}$

3. Solve the following on the interval $[0, 2\pi)$.

(a) $\sin(2\theta) + \sin(4\theta) = 0$

$\sin(4\theta) = \sin(2\cdot 2\theta) \overset{\alpha = 2\theta}{=} \sin(2\alpha)$

$= 2\sin\alpha\cos\alpha = 2\sin(2\theta)\cos(2\theta)$

since everything has argument $2\theta$ now, we can solve

$\sin(2\theta) + 2\sin(2\theta)\cos(2\theta) = 0$

$\sin(2\theta)(1 + 2\cos(2\theta)) = 0$

$\sin(2\theta) = 0$ or $1 + 2\cos(2\theta) = 0$

$\begin{cases} 2\theta = 0 + 2k\pi \\ 2\theta = \pi + 2k\pi \end{cases}$ or $\begin{cases} 2\theta = \frac{2\pi}{3} + 2k\pi \\ 2\theta = \frac{4\pi}{3} + 2k\pi \end{cases}$

$\Rightarrow \begin{cases} \theta = 0 + k\pi \\ \theta = \frac{\pi}{2} + k\pi \end{cases}$ or $\Rightarrow \begin{cases} \theta = \frac{\pi}{3} + k\pi \\ \theta = \frac{2\pi}{3} + k\pi \end{cases}$

Test k-values to find:

$\boxed{\theta = \left\{0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}}$

(b) $2\cos^2\theta + \cos\theta - 1 = 0$ quadratic!

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta - 1 = 0$ or $\cos\theta + 1 = 0$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ $\qquad \theta = \pi$

$\boxed{\theta = \left\{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}}$

(c) $\sin(2\theta) - \cos\theta - 2\sin\theta + 1 = 0$

$2\sin\theta\cos\theta - \cos\theta - 2\sin\theta + 1 = 0$

$\cos\theta(2\sin\theta - 1) - 2\sin\theta + 1 = 0$

$\cos\theta(2\sin\theta - 1) - (2\sin\theta - 1) = 0$

$(\cos\theta - 1)(2\sin\theta - 1) = 0$

$\cos\theta - 1 = 0$ or $2\sin\theta - 1 = 0$

$\theta = 0$ $\qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\boxed{\theta = \left\{0, \frac{\pi}{6}, \frac{5\pi}{6}\right\}}$