Chapter 2
Probability

Brittany Cuchta
STAT 3115, Spring 2015

Example 1

Consider a string of Christmas lights. If at least one of these lights fail, then the whole string will not light up.

We will denote a light failing by F and not failing by S. Let A be the event that the string does not light up. Possible outcomes in A include $FSSSSS$, $SFSSSS$, $SSFSSS$, etc. But it *also* includes simple events such as $FFSSSS$, $FFFSSS$, etc. There are a total of 63 outcomes in A. However, A', the event that the system works, has only *one* outcome, $SSSSSS$.

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Suppose the chance that any one light does not fail is 90%. If the components fail independently of each other (that is, one failing does not say anything about another light failing) then what is the probability of the system failing?

$$P(A) = 1 - P(A')$$

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\begin{aligned}
P(A) &= 1 - P(A') \\
&= 1 - P(SSSSSS) \\
&= 1 - 0.9^6 \qquad \textit{(We will see why this is the case in §2.5)}
\end{aligned}
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$$\begin{aligned} P(A) &= 1 - P(A') \\ &= 1 - P(SSSSSS) \\ &= 1 - 0.9^6 \quad \textit{(We will see why this is the case in §2.5)} \\ &= 1 - 0.531 \\ &= 0.469 \end{aligned}$$

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☆ One of the key phrases here was "at least." The complement of this phrase would be "less than" and is often much easier to deal with.

Example 2

Let E be the event that a car requires engine work under warranty and T be the event that the car requires transmission work under warranty. Suppose we know the following probabilities:

$$P(E) = 0.1 \quad P(T) = 0.02 \quad P(E \cap T) = 0.01$$

- Find the probability that the car needs work on the engine, the transmission, or both.
- Find the probability that neither the transmission nor the engine needs work.

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$$P(E' \cap T') = P\big[(E \cup T)'\big]$$

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Example 3

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There are $3 \cdot 4 \cdot 4 \cdot 2 = 96$ different computers.

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Example 5

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- How many ways could we rearrange the entire alphabet (all 26 characters)?

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- How many ways could we rearrange the entire alphabet (all 26 characters)?

$$P_{26,26} = \frac{26!}{0!} = 4.0329146 \times 10^{26}$$

Example 6

Suppose I need to enlist students' help in writing problems on the board, as in Example 4. I still have 42 students, but now, instead of giving them different problems, I'm going to give them all the same problem (unbeknownst to them). How many ways could I select the six students?

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Unlike last time, it does not matter what order students are selected in—they will all work the same problem. So, I will only need to use combinations.

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$$\binom{42}{6} = \frac{42!}{6!(42-6)!}$$

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Example 7

I would like to start a Discrete Mathematics Club. To do so, I need to enlist the membership of some faculty at the school. I decide that I will have 4 faculty from the maths department and 3 from the computer science department. How many ways can I select the faculty members if there are 12 maths faculty and 11 CS?

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The key word here is "and." I need 4 maths *and* 3 CS. I'll need to use multiplication!

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$$\binom{12}{4} \cdot \binom{11}{3} = 81,675$$

Example 8

I decide to flip a fair coin three times in a row. What's the probability I will get *exactly* 2 heads? What's the probability I will get *at least* 2 heads?

First, for exactly 2 heads, I will use the simple $\frac{N(A)}{N}$ idea, where A is the event I get 2 heads.

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$$\begin{aligned} P(\text{exactly 2 heads}) &= \frac{\text{number of ways to get 2 heads}}{\text{total number of possibilities}} \\ &= \frac{\binom{3}{2}}{2^3} \\ &= 0.375 \end{aligned}$$

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$$\begin{aligned} P(\text{at least 2 heads}) &= \frac{\text{number of ways to get 2 heads } \textit{or} \text{ 3 heads}}{\text{total number of possibilities}} \\ &= \frac{\binom{3}{2} + \binom{3}{3}}{2^3} \end{aligned}$$

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Does this surprise you?

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$$P(\text{heads on the second flip}) = \frac{2 \cdot 1 \cdot 2}{2^3} = 0.5$$

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$$P(\text{heads on first or second flip}) = \frac{1 \cdot 2 \cdot 2 + 2 \cdot 1 \cdot 2}{2^3} = 0.75$$

Example 10

A sample of 10 nurses is to be selected from a total of 90 employed by a hospital. Of the 90 nurses, 20 of them are male. What is the probability exactly 4 members of the sample will be male? At least 4?

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$$P(4 \text{ males and } 6 \text{ females}) = \frac{\binom{20}{4} \cdot \binom{70}{6}}{\binom{90}{10}} = 0.111$$

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$$P(4,\ 5,\ 6,\ 7,\ 8,\ 9, \text{ or } 10 \text{ males}) = 1 - P(0,\ 1,\ 2, \text{ or } 3 \text{ males})$$

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For at least 4 males,

$$\begin{aligned} P(4, 5, 6, 7, 8, 9, \text{or } 10 \text{ males}) &= 1 - P(0, 1, 2, \text{or } 3 \text{ males}) \\ &= 1 - \frac{\binom{20}{0}\binom{70}{10} + \binom{20}{1}\binom{70}{9} + \binom{20}{2}\binom{70}{8} + \binom{20}{3}\binom{70}{7}}{\binom{90}{10}} \end{aligned}$$

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Example 11

Sally has a lemonade stand. On a given day, she collected the following information about her customers.

	Small	Regular	Large	
Adult	20	10	3	
Child	2	5	15	

- Find the probability a randomly selected customer orders a small lemonade.
- Find the probability that a child customer orders a large lemonade.
- Given that the customer orders a small, what is the probability that the customer is an adult?

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Sally has a lemonade stand. On a given day, she collected the following information about her customers.

	Small	Regular	Large	
Adult	20	10	3	33
Child	2	5	15	22
	22	15	18	

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- Find the probability that a randomly selected customer orders a small lemonade.

$$P(\text{small}) = \frac{22}{55} = 0.4$$

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	22	15	18	55

- Find the probability that the customer is a child and orders a large lemonade.

$$P(\text{large lemonade and ordered by child}) = \frac{15}{55} = 0.\overline{27}$$

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- Given that the customer orders a small, what is the probability that the customer is an adult?

$$P(\text{adult} \mid \text{small}) = \frac{P(\text{small lemonade and ordered by adult})}{P(\text{small})}$$

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$$P(\text{adult} \mid \text{small}) = \frac{P(\text{small lemonade and ordered by adult})}{P(\text{small})}$$

$$= \frac{\frac{20}{55}}{\frac{22}{55}}$$

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$$\begin{aligned} P(\text{adult} \mid \text{small}) &= \frac{P(\text{small lemonade and ordered by adult})}{P(\text{small})} \\ &= \frac{\frac{20}{55}}{\frac{22}{55}} \\ &= 0.\overline{90} \end{aligned}$$

Example 12

A news magazine publishes three columns entitled "Art" (A), "Books" (B), and "Cinema" (C). Reading habits of regular subscribers with respect to these columns are

Reads Regularly	A	B	C	$A \cap B$	$A \cap C$	$B \cap C$	$A \cap B \cap C$
Probability	0.10	0.25	0.30	0.08	0.05	0.20	0.03

- Find the probability that a subscriber reads only A.
- Find the probability that a subscriber reads A and B but not C.
- Given that a subscriber reads at least one column, what's the probability that they read A?
- Given that a subscriber reads C, what's the probability that they read A, or B, or both?

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$$P(A \cap B' \cap C') = 0$$

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- Find the probability that a subscriber reads A and B but not C.

$$P(A \cap B \cap C') = 0.05$$

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Probability	0.10	0.25	0.30	0.08	0.05	0.20	0.03

- Given that a subscriber reads at least one column, what's the probability that they read A?

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$$P(A \mid \text{at least one}) = \frac{P(A \cap (A \cup B \cup C))}{P(A \cup B \cup C)}$$

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- Given that a subscriber reads C, what's the probability that they read A, or B, or both?

$$P(A \cup B \mid C) = \frac{P((A \cup B) \cap C)}{P(C)} = \frac{0.02 + 0.03 + 0.17}{0.30} = 0.73$$

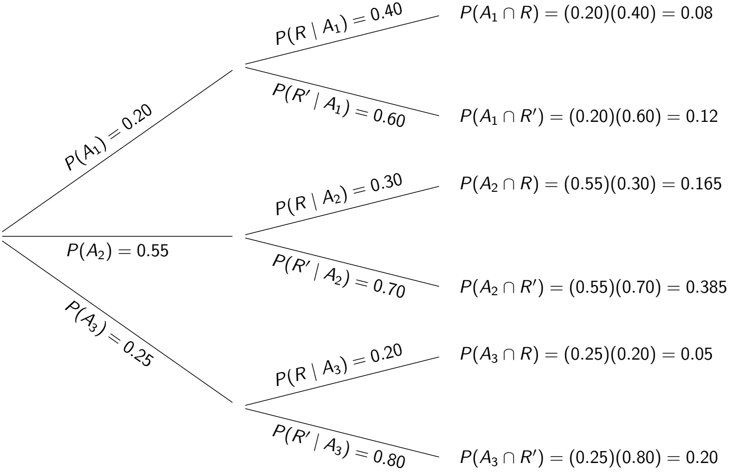
Example 13

The local electronics store has taken an interest in the likelihood of repairs for each of the three types of cell phones it sells. Of its sales, 20% of people buy Brand 1, 55% of people buy Brand 2, and the remaining 25% of people buy Brand 3. They have data suggesting that 40% of Brand 1 requires repair work within one year, 30% of Brand 2 requires work, and 20% of Brand 3 requires work.

- What is the probability a randomly selected purchaser will need repair work on the phone?
- Suppose a customer returns to the store and needs repair work on his phone. What is the probability that it is Brand 1?

Example 13

Use a tree diagram to find all necessary probabilities.



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P(R) &= P[(A_1 \cap R) \cup (A_2 \cap R) \cup (A_3 \cap R)] \\
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$$P(A_1 \mid R) = \frac{P(A_1 \cap R)}{P(R)}$$
$$= 0.388$$

Example 14

Suppose we have two hats: Hat 1 has 4 red balls and 6 green balls, and Hat 2 has 6 red balls and 4 green balls. We toss a fair coin, with heads indicating we will draw from Hat 1, and tails indicating we will draw from Hat 2. What is the probability of getting a red ball?

Example 14

Suppose we have two hats: Hat 1 has 4 red balls and 6 green balls, and Hat 2 has 6 red balls and 4 green balls. We toss a fair coin, with heads indicating we will draw from Hat 1, and tails indicating we will draw from Hat 2. What is the probability of getting a red ball?

$$P(\text{Red}) = P(\text{Red} \mid H)P(H) + P(\text{Red} \mid T)P(T)$$

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☆ Notice that Heads and Tails are mutually exclusive and exhaustive events for the flipping of a coin! The total probability of drawing a red ball is a weighted probability, with the weights being the probability of getting heads or tails.

Example 15

Only 1 in 1000 of adults is afflicted with a rare disease. A new diagnostic test has been developed. When an individual has the disease, a positive result will occur 99.5% of the time and an individual without the disease will show a positive result only 1% of the time. If a randomly selected individual is tested and has a positive result, what is the probability that the individual has the disease?

Example 16

Oil wells drilled in Region A have a 0.2 probability of producing and wells from Region B have a probability of 0.09 of producing. Assume the regions produce independently.

- Find $P(A \cap B)$.
- Find $P(A \cup B)$.
- Find $P(A' \cap B')$.

Example 16

Oil wells drilled in Region A have a 0.2 probability of producing and wells from Region B have a probability of 0.09 of producing. Assume the regions produce independently.

- Find $P(A \cap B)$.

$$\begin{aligned} P(A \cap B) &= P(A)P(B) \\ &= 0.2(0.09) \\ &= 0.018 \end{aligned}$$

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- Find $P(A \cup B)$.

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.2 + 0.09 - 0.018 \\ &= 0.272 \end{aligned}$$

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- Find $P(A' \cap B')$.

$$\begin{aligned} P(A' \cap B') &= P[(A \cup B)'] \\ &= 1 - P(A \cup B) \\ &= 1 - 0.272 \\ &= 0.728 \end{aligned}$$