Chapter 3
Discrete Random Variables and Probability Distributions

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STAT 3115, Spring 2015

Example 1

A mail-order computer business has six telephone lines. Let X denote the number of lines in use at a specified time. Suppose the pmf of X is as given in the table below.

x	0	1	2	3	4	5	6
$p(x)$	0.10	0.15	0.20	0.25	0.20	0.06	0.04

Calculate the following probabilities:

- {at most three lines are in use}
- {fewer than three lines are in use}
- {at least three lines are in use}
- {between two and five lines, inclusive, are in use}

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Calculate the following probabilities:

- {at most three lines are in use}

$$P(X \leq 3) = p(0) + p(1) + p(2) + p(3) = 0.10 + 0.15 + 0.20 + 0.25 = 0.70$$

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Calculate the following probabilities:

- {fewer than three lines are in use}

$$P(X < 3) = p(0) = p(1) + p(2) = 0.45$$

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Calculate the following probabilities:

- {at least three lines are in use}

$$P(3 \leq X) = p(3) + p(4) + p(5) + p(6) = 0.55$$

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Calculate the following probabilities:

- {between two and five lines, inclusive, are in use}

$$P(2 \leq X \leq 5) = p(2) + p(3) + p(4) + p(5) = .71$$

Example 2

The pmf of the amount of memory X (GB) in a purchased flash drive is given as

x	1	2	4	8	16
$p(x)$	0.05	0.10	0.35	0.40	0.10

Find $E(X)$.

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$$\begin{aligned} E(X) &= \sum_{x=1}^{16} x \cdot p(x) \\ &= 1(0.05) + 2(0.10) + 4(0.35) + 8(0.40) + 16(0.10) \end{aligned}$$

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E(X) &= \sum_{x=1}^{16} x \cdot p(x) \\
&= 1(0.05) + 2(0.10) + 4(0.35) + 8(0.40) + 16(0.10) \\
&= 6.45
\end{aligned}
$$

Example 3

An individual who has auto insurance from a certain company is selected. Let Y be the number of moving violations for which the individual was cited during the last three years. The pmf of Y is

y	0	1	2	3
$p(y)$	0.60	0.25	0.10	0.05

- Find $E(Y)$.
- Suppose an individual with Y violations incurs a surcharge of \$$100Y^2$. Calculate the expected amount of the surcharge.

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$p(y)$	0.60	0.25	0.10	0.05

- Find $E(Y)$.

$$E(Y) = \sum_{x=0}^{3} y \cdot p(y) = 0(0.6) + 1(0.25) + 2(0.10) + 3(0.05) = 0.60$$

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\begin{aligned}
E(100Y^2) &= \sum_{x=0}^{3} 100y^2 \cdot p(y) \\
&= 100(0)(0.60) + 100(1)(0.25) + 100(4)(0.10) + 100(9)(0.05)
\end{aligned}
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&= 100(0)(0.60) + 100(1)(0.25) + 100(4)(0.10) + 100(9)(0.05) \\
&= 110
\end{aligned}
$$

Example 4

The pmf of the amount of memory X (GB) in a purchased flash drive is given as

x	1	2	4	8	16
$p(x)$	0.05	0.10	0.35	0.40	0.10

Find the variance and standard deviation of X.

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Find the variance and standard deviation of X.

$$V(X) = E(X^2) - [E(X)]^2 = \sum_{x=1}^{16} x^2 \cdot p(x) - 6.45^2$$

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$$\begin{aligned} V(X) = E(X^2) - [E(X)]^2 &= \sum_{x=1}^{16} x^2 \cdot p(x) - 6.45^2 \\ &= 1(0.05) + 4(0.10) + 16(0.35) + 64(0.40) + 256(0.10) - 6.45^2 \end{aligned}$$

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The pmf of the amount of memory X (GB) in a purchased flash drive is given as

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$p(x)$	0.05	0.10	0.35	0.40	0.10

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&= 1(0.05) + 4(0.10) + 16(0.35) + 64(0.40) + 256(0.10) - 6.45^2 \\
&= 15.6475
\end{aligned}
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$p(x)$	0.05	0.10	0.35	0.40	0.10

Find the variance and standard deviation of X.

$$V(X) = E(X^2) - [E(X)]^2 = \sum_{x=1}^{16} x^2 \cdot p(x) - 6.45^2$$

$$= 1(0.05) + 4(0.10) + 16(0.35) + 64(0.40) + 256(0.10) - 6.45^2$$

$$= 15.6475$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{15.6475} = 3.956$$

Example 5

Suppose that 30% of all students who have to buy a text for a particular course want a new copy, whereas the other 70% want a used copy. Consider randomly selecting 25 purchasers.

- What are the mean value and SD of the number who want a new copy of the book?
- What is the probability that the number who want new copies is more than two SDs away from the mean?
- The bookstore has 15 new copies and 15 used copies in stock. If 25 people come in one by one to purchase this text, what is the probability that all 25 will get the type of book that they want from the current stock?
- Suppose that new copies cost \$100 and used copies cost \$70. Assume the bookstore currently has 50 new copies and 50 used copies. What is the expected value of total revenue from the sale of the next 25 copies purchased?

Example 6

An electronics store has received a shipment of 20 table radios that have connetions for an iPod or iPhone. Twelve of these have two slots (so they can accomodote both devices) and the other eight have a single slot. Suppose that six of the 20 radios are randomly selected to be stored under a shelf where the radios are displayed, and the remaining placed in a storeroom. Let X be the number among the radios placed on display that have two slots.

- What kind of distribution does X have?
- Compute $P(X = 2)$, $P(X \leq 2)$, $P(X \geq 2)$.
- Calculate the mean and SD of X.

Example 7

Each of 12 refrigerators of a certain type has been returned to a dstributor because of an audible, high-pitched, oscillating noise when the refrigerators are running. Suppose that 7 of the refrigerators have a defective compressor and the other 5 have less serious problems. If the refrigerators are examined in random order, let X be the number among the first 6 examined that have a defective compressor.

- Find $P(X = 5)$.
- Consider a large shipment of 400 refrigerators, of which 40 have defective compressors. If X is the number among 15 randomly chosen refrigerators to have defective compressors, approximate $P(X \leq 5)$.

Example 8

Suppose that $p = P(\text{male birth}) = 0.5$. A couple wishes to have exactly two female children in their family and will continue to have children until this condition is fulfilled.

- What is the probability that the family has four children?
- What is the probability that the family has at most four children?
- How many male children would you expect this family to have? How many children would you expect this family to have?

Example 9

Let X, the number of flaws on the surface of a randomly selected boiler of a certain type, have a Poisson distribution with parameter $\mu = 5$. Find the following probabilities:

- $P(X = 8)$
- $P(X \leq 8)$
- $P(9 \leq X)$
- $P(5 \leq X \leq 8)$
- $P(5 < X < 8)$